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Regression Modeling of Thermo–Spartial Properties of Shell and Tube Heat Exchanger

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ABSTRACT

Thermo – Spatial properties directly influence the rate of heat transfer and overall performance of heat exchanger systems. It is imperative to understand how changes in the thermo – spatrial conditions impact on the performance of HE system for optimal design considerations through system's behavoural prediction. Mathematical models have continued to be useful in understanding such relationships and influenced design decisions significantly. This research probed the ansys simulated data from (Saad et al, 2022) to derive a mathematical model for the rate of heat transfer with respect to length of exchanger and logarithmic mean temperature difference for a counter flow exchanger system using Yule's multiple regression analytic method. Only four lengths (10m – 25m) of the heat exchanger were considered in their work with corresponding rate of heat transfer values in kilowatts. However, for data symmetry considerations, two other lengths (5m and 30m) were included by means of linear extrapolation in the lower and upper extremes of heat exchanger lengths along with their logarithmic meant temperature difference (LMTD) and rate of heat transfer values. The model obtained indicates good approximation for the data as the mean sum of squares of the errors (MSSE) due to various sources were analytically obtained to be minimum. The model showed steady increase in the rate of heat transfer with increase in the length of exchanger at the rate of 0.18kw for every 1m change in length and a 1.06kw change in the heat transfer for every 1K rise or fall in the logarithmic mean temperature difference (LMTD). The model also revealed that the temperature of the cold fluid never rose to the lowest temperature of hot fluid, this implies that the logarithmic mean temperature difference cannot be extrapolated to an absolute zero, invariably indicating that the system never reached thermal equilibrium and therefore, Zeroth and the Third Laws of Thermodynamics were not violated and heat exchange was continuous in the system at a mean value of 60.9kw. The coefficient of multiple determination was obtained as $(R^2_{Q.1\Delta\theta_m} = 0.9995)$, suggesting that about 99.95% of the variations in the heat transfer are due to changes in the thermo – spatial properties in the HE. Analysis of variance of model showed that $F_{cal} > F_{critical}$ @ $\propto = 0.05$ which rejects the null hypothesis and accepts the alternative hypothesis that there is significant impact of thermo – spatial properties on heat transfer.

1. Introduction

Heat exchangers are some of the critical components of engineering systems and processes. The performance of heat exchangers can directly or indirectly affect the overall performance of a system. Heat exchangers are utilized as sub units in larger systems to ensure thermal balance and improved efficiency to the entire system. Hence, an efficiently performing heat exchanger arrangement could mean an optimally performing system. The choice of heat exchanger employed in any system depends on the purpose and energy requirement of the system. Heat exchangers convert thermal energy to electrical energy in a system as well as maintaining thermal balance throughout the system by means of thermal exchange of fluid flow within and around arrangement of tubes. For every heat exchanger arrangement, the main aim is to improve the heat transfer rate and efficiency while minimizing pressure losses. Heat exchangers are manufactured as; heat recovery steam generators, recuperators, regenerators, intercoolers, aftercoolers etc. These heat exchangers are incorporated in various systems such as; heating, ventilation, air-conditioning (HVAC) systems, aerodynamic and automotive systems, air compressors, cryogenic systems, internal combustion engines, power plant system such as turbines, marine system etc. The regenerator and recuperator types are designed to collect and transfer hot gas especially from turbine exhaust systems and convey it to the discharge air from the compressor, leading to improved efficiency of both simple and complex energy generating systems. Cooling system devices such as the air-compressor; oil coolers, intercoolers, aftercooler type exchangers are used to reduce and cool the temperature of air from the compressor. In general, the type of application determines the design style, materials choice, fluids, tube dimensions and spacing of the heat exchanger to be used.

It is imperative to explore system data to establish mathematical models relating thermo – spatial properties that can be representative of heat exchanger systems and predict with certainty the amount of heat transfer and/or efficiency of particular type heat exchangers with respect to critical input variables from experimentally acquired or simulated data. Literature reveals the dependence of the heat transfer rate on several input variables of heat exchanger systems. Hence, statistical models relating multivariate systems such as the heat exchanger hold so much promise in solving the problem of establishing deterministic models for the heat transfer, efficiency and other output parameters for particular type heat exchangers for given empirical data. A representative model of the exchanger system will not only equip design engineers with good knowledge of the combination of input variables that maybe required to achieve specific amount of heat transfer but will also optimize design, enhance system sustainability, cost saving, improve efficiency of the entire system as well as assist thermal process software engineers and developers to integrate the model for prediction and optimization of thermal condition of the system at all times.

The choice of design parameters used for a particular type of heat exchanger including inlet and outlet conditions of both fluids (hot and cold) is critical to the overall performance of its applied system in terms of heat transfer rate as well as energy efficiency. The extreme temperatures of the working fluids, their properties, fluid flow rates are the critical design inputs for a heat exchanger [1].

The most critical thermo – spatial parameters in the design and operation of heat exchangers are the inlet temperature and velocity of the hot and cold fluids, the outlet temperature, the tube lengths and diameters, as well as type of working fluids. These parameters greatly influence the temperature distribution, pressure drop, rate of heat transfer and the efficiency of the heat exchanger. These variations can be experimentally determined or simulated using software with computational fluid dynamics capabilities by testing a set of carefully selected variables and boundary conditions known from theoretical background for particular heat exchanger design. The observations from such experimental works can be further probed to establish intrinsic patterns that could lead to insightful mathematical relationships between input and output variables. The extent of reliability or strength of the relationship can for be investigated with more sophisticated statistical tools and methods to understand the degree to which such relationships hold true according to given observations.

1.1 Types of Heat Exchangers

Heat exchangers are of various types and are classified according to; structural outlook, number of working fluids, transfer process, heat transfer mechanism, flow pattern and surface compactness. However, we shall restrict our discussion to the structural outlook and flow pattern categories.

Structural outlook: there are four main types that constitute this category viza-viz; tubular heat exchangers, plate type heat exchangers, regenerative heat exchangers and extended surface heat exchangers.

Tubular exchangers: these are primarily built of elliptical, circular, round/flat twisted tubes. They are usually designed to withstand high pressure differences between working fluids in the exchanger and its surrounding. These types of heat exchangers are used for gas-to-gas and gas-to-liquid heat transfer application particularly when the operating temperature/pressure or both is very high or if fouling is of a significant interest to be considered for any of the operating fluids and their surfaces in contact. They are also used for liquid-to-liquid and liquid-to-phase change heat transfer applications.

Tubular exchangers are further classified into shell-and-tube exchangers, double-pipe exchangers and spiral tube exchangers. Care must be taken to understand that baffle exchangers, square/rectangular exchangers and triangular exchangers are subdivisions of the shell-and-tube type exchangers and are so named according arrangement of how sheets, rods and baffles are used to hold the tubes in position in the heat exchanger.

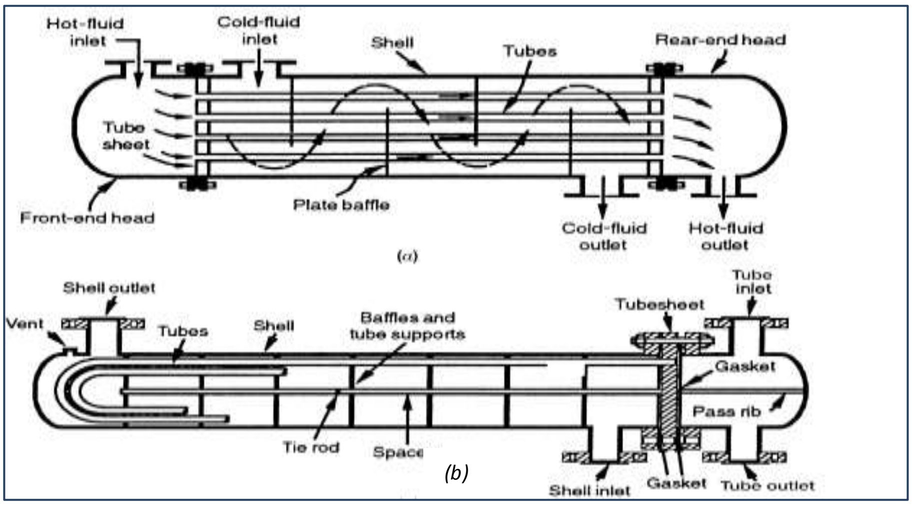

Figure 1 (a) A 2D schematic of a shell-and-tube exchanger with one shell and one tube passes; 1 (b) A 2D schematic of a shell-and-tube exchanger with one shell and two tube passes [1].

2. Literature Review

Here we explore existing research outcomes from related subjects and establish a gap to justify the motivation behind our research. There are exhaustive number of academic and industrial works carried out on the design and evolution of heat exchangers ranging from conceptual designs, manufacturing methods, optimization and its application to various industrial systems such as in aerodynamics, automotive, power plant systems, heavy duty factory set-ups, HVAC systems etc. Some of these works are considered in details in the following paragraphs to provide good background for the proposed model.

The effect of length of exchanger on the rate of heat transfer was investigated for both parallel and counter flows with water as liquid medium between the hot and cold fluids; lengths were varied from 10 – 25m in increment of 5m steps per trial and the rate of heat transfer was observed to be between 42.9kW and 50.2kW respectively for the parallel flow while the counter flow arrangement for same set of length variation revealed heat transfer rates to fall between 48.7kW and 69.6kW respectively which clearly shows that quality of heat transferred and overall efficiency for both parallel and counter flow exchangers increases with the length of exchanger. However, with the counter flow showing better heat transfer rates and efficiency. In both arrangements, temperature profiles indicate that temperature gaps are inversely proportional to the length of exchanger as well as heat transfer rate [1]. However, this work clear did not consider the effect of other factors like tube diameter and pitch of the tubes on the heat transfer rate and efficiency of the exchanger. The investigation also fail to establish a deterministic relationship between exchanger length and rate of heat transfer to precisely predict a specific quantity of the heat transferrable for any given length of the exchanger. The rate of heat transfer and pressure drop with respect to triangular and rectangular tube bundles arrangement were experimentally determined with Grimson and Zukauskas equations and same were compared with results of ansys fluent software for exact same set-up; the ratio of the distance between two tubes adjacent to each other in the direction of flow to the diameter of any one tube and the ratio of the distance between two adjacent tubes perpendicular to the direction of flow to the tube diameter for both triangular and rectangular tube bundles arrangement were used as control variables for their investigation. The output were investigated for ratios of 1.25 – 3 for both lateral and perpendicular arrangements and it was observed that with the perpendicular ratio fixed at 1.25 with increasing lateral ratio, the heat transfer decreased significantly for the Grimson experiment but showed opposite result for the ansys fluent simulation; however, increased with other fixed values of the perpendicular ratio with increasing lateral ratio shows that the results for both experimental and simulated values for the triangular tube bundle arrangement were completely opposite [2]. The research did not consider other important factors like material properties, flow type, flow rates of hot and cold fluid, boundary conditions, temperature distribution withing the heat exchanger and ultimately did not investigate the effect of rate of change of perpendicular and lateral ratios on the rate of heat transfer. The heat exchanger design is complicated and beyond just heat transfer analysis as it requires a good estimate of fabrication and installation costs, as well as size and weight considerations from ownership point of view [3]. This research did not quantitatively probe the variations of critical exchanger design parameters with respect to heat transfer rate and/or efficiency of the exchanger. Cost effect was not also probed beyond mere statements and certainly no type of quantifiable relationships were considered and established. The effect of the inner shape of double piped heat exchanger on the rate of heat transfer was probed for triangular, hexagonal and octagonal inner pipe shapes with inlet ethanol temperature at 78^o flowing through the inner pipes in each case while water at 10^o flows through the outer pipe at inlet with a flow rate of $0.1kg^{-s}$ and turbulent intensity of 5% of turbulent viscosity ratio of 10. Outer pipe was made of brass and inner pipe was made of copper throughout the investigation. Ansys CFD software was used to simulate the practical set-up for the heat transfer analysis for both dented and undented pipe patterns and it was discovered that heat transfer was enhanced with increase in the number of sides of the inner pipe for the undented inner pipe case while the dented inner pipe case gave optimal heat transfer rate for inner pip shapes [4]. This research fails to discuss the exact manufacturing processes suitable for the types of heat exchangers proposed in their work. The research did not also provide the numerical relationship existing between pipe dimensions (input) and rate of heat transfer (output). From investigations carried out on a vertical shell-and-tube exchanger with a complex geometry and complicated flow pattern comprising forty-nine (49) tubes of 865mm length with 28mm and 24mm in external and internal diameters respectively, using the Wilson plot method and its modification based on measured experimental data to determine the heat transfer coefficient on both the condensation and cooling sides of the exchanger. The condensation side assumed a constant thermal resistance value based on the original Wilson plot method but varying thermal resistance for the modified Wilson plot (valid for Nusselt theory for estimating the heat transfer coefficient) method due to different fluid flow conditions such as fluid velocity and temperature. Hence, a modified Wilson plot method that could improve the determination accuracy of the criterion equation for the heat transfer coefficient was proposed and it showed good agreement with the experimental results as well as commonly used theoretical methods [5]. Here, no mathematical relationships nor the effect of fouling of the shell and tube walls were considered. In the design and performance evaluation of a shell-and-tube exchanger with 21 tubes, 610mm shell length and 170mm diameter using Kern’s technique on the application of Solidworks software for the 3D model of the exchanger and Ansys CFD package simulation. The heat exchanger was modified from a water flowing fluid shell side to a methanol fluid shell side and it was observed that the effectiveness but heat transfer rate was not uniform throughout the length of the exchanger [6]. Once again, a deterministic model relating the boundary conditions (design parameters) and exchanger effectiveness was ignored in this investigation. Exhaustive study of the various types of heat exchangers and their variants in existence since time, their specific areas of and conditions of application, optimization of various types in terms of geometry, design parameters and detailed explanation of the theoretical principles, equations governing each type and their unique thermo-fluid concepts as well as experimental investigations of critical parameters and simulation of some of the types of exchangers covered using Ansys fluent software have been carried out by [7]. Here, very little effort was made at undertaking quantitative analysis for the various types of heat exchangers explained.

3.0 Methodology

This research is founded on the works of [1] carried out on a shell-and-tube exchanger for a counter flow and summary of their work has been uncovered in the literature review section of this research. Furthermore, the Newton-Gregory method shall be employed to linear extrapolate more data points for analysis and model generation according to [8]. The concept of multiple correlation and regression shall be the core of the mathematical modeling to measure the amount of linear relationship existing between the input and output variables. Matlab software shall be utilized to generate a scatter plot for the plane of multiple regression for the acquired data for the work. Finally, an F-test is carried out to ascertain the strength of the established relationship through the instrument of Analysis of Variance (anova) for the data set.

3.1 Conceptual Framework of the Model

The conceptual framework of the model adopted for the research as shown in figure 3.1 tends to provide the flow of the system activities relating the type of heat exchanger, the critical explanatory and response variables that are relevant to the research, as well as performance of the entire system.

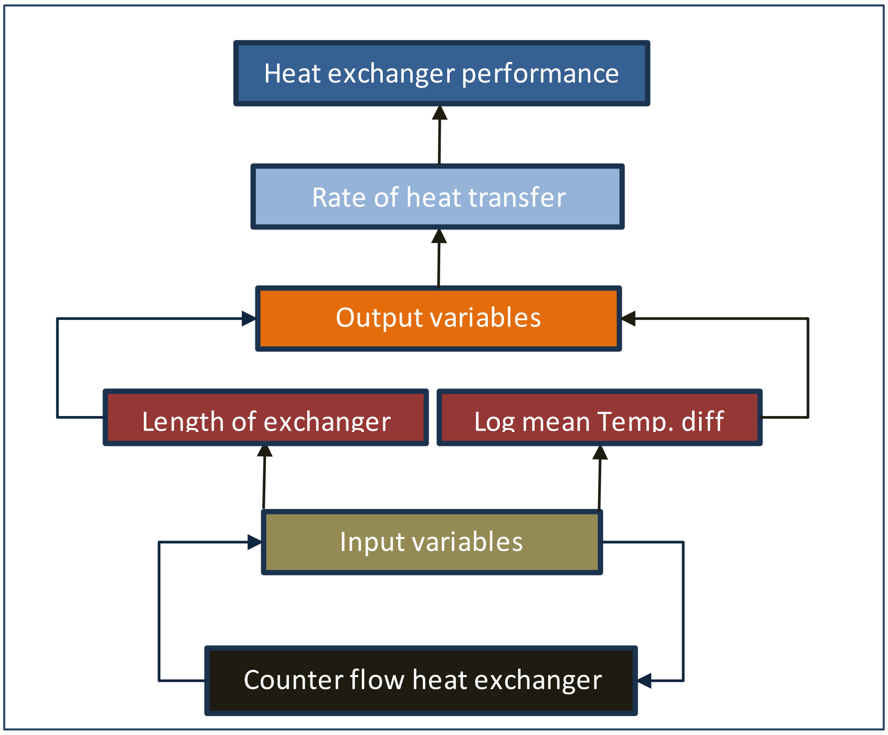

Figure 3.1 Conceptual framework of the model

3.2 Theory Of Correlation And Regression

The Theory of correlation assumes that for any statistical inquiry, the observations obtained contain underlying features, patterns and trends that can be measured linearly to certain degree of confidence. The process of deriving the model that best approximate the relationship between the explanatory and response variables for a given set of data is called regression fitting. Investigations involving a bivariate data is called a simple correlation/regression analysis while those involving multivariate data are referred to as multiple correlation/regression analysis and this study and this research rightly falls under the latter category as we intend to investigate the variations in the rate of heat transfer due to changes in the length (l) of the exchanger and the logarithmic mean temperature difference$(\Delta\theta_m)$ across the tubes for a counter flow system through mathematical modeling.

3.3 Change of Origin

This is a powerful theory in correlation which states that the increase or decrease in the observations of given variables in a data does not affect the correlation coefficient of between the pairs of variables under consideration. Hence, by this theory; for a bivariate data of variables x_i *and* x_j; the change in origin in both variables by quantities (p) and (μ) is given $(x_i \pm p)$ and $(x_j \pm \mu)$ respectively.

However, it is pertinent to understand that the principles of correlation for a bivariate distribution lays the foundation for the analysis of a multivariate distribution. Now according to the theory of correlation, for a bivariate distribution x_i and x_j; the total correlation coefficient (r_{ij}) between the two variables is given by;

$$r_{(x_i \pm p)(x_j \pm \mu)} = r_{ij} = \frac{Cov\,(x_i,x_j)}{\sigma_i \sigma_j} \qquad (3.1)$$

Where $i \neq j = 1, 2, 3, \dots$ n

$\sigma_i \Rightarrow$ standard deviation of variable x_i; $\sigma_j \Rightarrow$ standard deviation of variable x_j and $Cov(x_i,x_j) \Rightarrow$ covariance of variables x_i and x_j.

Sticking with our research variables; length of exchanger (l), logarithmic mean temperature difference $(\Delta\theta_m)$ and rate of heat transfer (Q); the correlation coefficient between any pair of input variables becomes:

(i) *rate of heat transfer* (Q) *and length of exchanger* (l)

$$r_{Q,l} = \frac{Cov\,(Q,l)}{\sigma_Q\,\sigma_l} \quad (3.2)$$

(ii) *rate of heat transfer* (Q) *and logarithmic mean temperature difference* $(\Delta\theta_m)$ (3.3)

$$r_{Q,\Delta\theta_m} = \frac{Cov\,(Q,\Delta\theta_m)}{\sigma_Q\,\sigma_{\Delta\theta_m}}$$

(i) *Length of exchanger* (l) *and logarithmic mean temperature difference* $(\Delta\theta_m)$

$$r_{l,\Delta\theta_m} = \frac{Cov\,(l,\Delta\theta_m)}{\sigma_l\,\sigma_{\Delta\theta_m}} \quad (3.4)$$

From the works of Karl Pearson, derived versions of $(3.2) - (3.4)$ with application of the principle of change of origin are given in $(1iv)$.

$$\mathrm{r}_{Q,l} = \frac{\sum(Q-\bar{Q})(Q-\bar{Q})}{\sqrt{\sum(Q-\bar{Q})^2\ \sum(l-\bar{l})^2}} \quad (3.5)$$

However, for the current issue at hand, the change of origin in each variable is equivalent to their respective deviations from the mean as the quantity by which each variable has been considered to change its origin is equal to their respective means, and therefore (3.5) can further be written as:

$$\mathrm{r}_{Q,l} = \frac{\sum dQdl}{\sqrt{\sum dQ^2\ \sum dl^2}} \quad (5.6)$$

Similarly, (5.3) and (5.4) can as well be written as:

$$\mathrm{r}_{Q,\Delta\theta_m} = \frac{\sum dQd\Delta\theta_m}{\sqrt{\sum dQ^2\ \sum d\Delta\theta_m{}^2}} \quad (3.7)$$

$$\mathrm{r}_{l,\Delta\theta_m} = \frac{\sum dld\Delta\theta_m}{\sqrt{\sum dl^2\ \sum d\Delta\theta_m{}^2}} \quad (3.8)$$

3.4 Multiple Regression

The idea of multiple regression is rooted in the reality that no one factor alone is completely responsible for the observable variations in a known response variable, that a multiplicity of factors are usually responsible for the observed variations in a responsible variable. Hence, there is need to be able quantify these input variables alongside their associated response variable for the understanding of the behavoural patterns observed in the response variable for decision making. It is imperative to understand that the graph of the multiple regression is not a straight line and neither does it even represent series of parallel line as measures of the number of input variables but it is a plane containing all variables involved. Hence, every multiple regression problem involves two or more explanatory variables with a single response variable. The general form of the plane of multiple regression is as given in (3.9).

$$X_1 = \beta_o + \beta_1 X_2 + \beta_2 X_3 + \cdots + \beta_k X_n + \varepsilon \quad (3.9)$$

Where: $X_1 \Rightarrow$ the response variable; $X_2, X_3, \dots X_n \Rightarrow$ the various explanatory variables of the data;

$\beta_o \Rightarrow$ the multiple regression constant (which is the minimum value for the response variable in the model);

$\beta_1 \Rightarrow$ the first multiple regression coefficient and it is the rate at which the response variable changes with input variable X_2; $\beta_2 \Rightarrow$ the second multiple regression coefficient and it is the rate at which the response variable changes with input variable X_3 and $\varepsilon \Rightarrow$ the error term of the model. There are a couple of methods for fitting the regression model for a given data set but we shall only adopt the Prof. Yul's method which was built upon the principle of the least square regression model fitting.

3.5 Yule's Multiple Regression Model

Yul's method of fitting the plane of regression for a given set of data is built on the principles of least square method of regression fitting. The principle states that the plane of multiple regression that best approximate a given data is the one in which the sum of squares of the errors is least or minimum.

However, before we make further progress with principle of Least Squares, let us rewrite (5.9) to reflect our research variables in use. Since our proposed model is a trivariate system, hence, we have:

$$Q = \beta_o + \beta_1 l + \beta_2 \Delta\theta_m \quad (3.10)$$

Observe that the error term (ε) is dropped in (3.10). This is because it can be independently determined using the residuals of the fitted model without necessarily affecting the entire model and sometimes quite negligible to be considered.

A fundamental assumption of the Yul's method is that the observations of each variable are measured from their respective means. This key assumption is validated by the theory of change of origin which has been established in section 3.3.1 according to [10]. According Yul's notations, the plane of multiple regression as given in (5.10) can as well be written as:

$$Q = \beta_o + \beta_{Ql.\Delta\theta_m} l + \beta_{Q\Delta\theta_m.l} \Delta\theta_m \quad (3.11)$$

Each parameter retains its original meaning. Since the key assumption states that the observations of the trivariate distribution are measured from their respective means, it therefore means that the mathematical expectation of the variables equals zero (obvious). This $\Rightarrow$ $[dQ = Q - \bar{Q};\ dl = l - \bar{l}$ and $d\Delta\theta_m = \Delta\theta_m - \overline{\Delta\theta_m}]$ and from the theory of change of origin, the mathematical expectations: $E(dQ) = E(dl) = E(d\Delta\theta_m) = 0$

Now rewriting (5.11) in terms of change of their respective origins we have:

$$dQ = \beta_o + \beta_{Ql.\Delta\theta_m} dl + \beta_{Q\Delta\theta_m.l} d\Delta\theta_m \Rightarrow \beta_o = \overline{dQ} - (\beta_{Ql.\Delta\theta_m} \overline{dl} + \beta_{Q\Delta\theta_m.l} d\overline{\Delta\theta_m})$$

$\Rightarrow \beta_o = E(dQ) - E(dl) - E(d\Delta\theta_m) = 0$. Hence, the Yul's multiple regression model is reduced to:

$$dQ = \beta_{Ql.\Delta\theta_m} dl + \beta_{Q\Delta\theta_m.l} d\Delta\theta_m \quad (3.12)$$

Recall from (1) that:

$$r_{Q,l} = \frac{Cov(Q,l)}{\sigma_Q \sigma_l} \Rightarrow Cov(Q,l) = r_{Q,l}\sigma_Q\sigma_l \quad (3.13)$$

But $Cov(Q,l) = \frac{1}{n}\sum dQl$ (3.14)

From (3.13) and (3.14) it $\Rightarrow \sum dQl = nr_{Q,l}\sigma_Q\sigma_l$ (3.15)

where n is the number of the pairs of observations for the variables. Arbitrarily dropping the n in equation (3.15) above becomes:

$$\sum dQl = r_{Q,l}\sigma_Q\sigma_l \quad (3.16)$$

And similar set of relationships can be derived between the rate of heat transfer and logarithmic temperature difference and between the length of exchanger and the logarithmic temperature difference as follows:

$$\sum dQ\Delta\theta_m = r_{Q,\Delta\theta_m}\sigma_Q\sigma_{\Delta\theta_m} \quad (3.17)$$

$$\sum dl\Delta\theta_m = r_{l\Delta\theta_m}\sigma_l\sigma_{\Delta\theta_m} \quad (3.18)$$

Similarly, their respective variances are given by: $\sigma_i^2 = \frac{\sum dx_i^2}{n} \Rightarrow \sum dx_i^2 = n\sigma_i^2$. Hence, similar expressions for the rate of heat transfer, length of exchanger and logarithmic temperature difference becomes:

$$\sum dQ^2 = \sigma_Q^2 \quad (3.19)$$

$$\sum dl^2 = \sigma_l^2 \quad (3.20)$$

$$\sum d\Delta\theta_m{}^2 = \sigma_{\Delta\theta_m}^2 \quad (3.21)$$

The respective variances of the variables are:

$$\sigma_Q^2 = \frac{\sum dQ^2}{n};\ \sigma_l^2 = \frac{\sum dl^2}{n};\ \sigma_{\Delta\theta_m}^2 = \frac{\sum d\Delta\theta_m{}^2}{n}$$

From (3.21) $dQ = \beta_{Ql.\Delta\theta_m} dl + \beta_{Q\Delta\theta_m.l} d\Delta\theta_m$ by principle of the least squares, the sum of squares of the errors (SSE) from the plane of regression is given by:

$$\sum E^2 = \sum\left(\beta_{Ql.\Delta\theta_m} dl + \beta_{Q\Delta\theta_m.l} d\Delta\theta_m - dQ\right)^2 \quad (3.22)$$

The minimum value of (3.22) is given by the partial derivatives of the SSE w.r.t the coefficients of the length exchanger and the logarithmic temperature difference. Therefore, we have:

$$\frac{\partial(\sum E^2)}{\partial\beta_{Ql.\Delta\theta_m}} = 2\sum dl\left(\beta_{Ql.\Delta\theta_m} dl + \beta_{Q\Delta\theta_m.l} d\Delta\theta_m - dQ\right) = 0$$

$$\Rightarrow \beta_{Ql.\Delta\theta_m}\sum dl^2 + \beta_{Q\Delta\theta_m.l}\sum dl\,\Delta\theta_m - dQl = 0$$

$$\Rightarrow dQl = \beta_{Ql.\Delta\theta_m}\sum dl^2 + \beta_{Q\Delta\theta_m.l}\sum dl\,\Delta\theta_m \quad (3.23)$$

$$\frac{\partial(\sum E^2)}{\partial\beta_{Q\Delta\theta_m.l}} = 2\sum dl\left(\beta_{Ql.\Delta\theta_m} dl + \beta_{Q\Delta\theta_m.l} d\Delta\theta_m - dQ\right) = 0$$

$$\Rightarrow \beta_{Ql.\Delta\theta_m}\sum dl\,\Delta\theta_m + \beta_{Q\Delta\theta_m.l}\sum d\,\Delta\theta_m{}^2 - dQ\Delta\theta_m = 0$$

$$\Rightarrow dQ\Delta\theta_m = \beta_{Ql.\Delta\theta_m}\sum dl\,\Delta\theta_m + \beta_{Q\Delta\theta_m.l}\sum d\,\Delta\theta_m{}^2 \quad (3.24)$$

Substituting (3.17) *and* (3.18) in (3.23) *and* (3.24) gives:

$$\Rightarrow r_{Q,l}\sigma_Q\sigma_l = \beta_{Ql.\Delta\theta_m}\sigma_l^2 + \beta_{Q\Delta\theta_m.l} r_{l\Delta\theta_m}\sigma_l\sigma_{\Delta\theta_m} \quad (3.25)$$

$$\Rightarrow r_{Q,\Delta\theta_m}\sigma_Q\sigma_{\Delta\theta_m} = \beta_{Q\Delta\theta_m.l}\sigma_{\Delta\theta_m}^2 + \beta_{Ql.\Delta\theta_m} r_{l\Delta\theta_m}\sigma_l\sigma_{\Delta\theta_m} \quad (3.26)$$

Solving (3.25) *and* (3.26) simultaneously gives:

$$\beta_{Ql.\Delta\theta_m} = \frac{\sigma_Q}{\sigma_l}\left(\frac{r_{Ql} - r_{Q\Delta\theta_m} r_{l\Delta\theta_m}}{1-(r_{l\Delta\theta_m})^2}\right) \quad (3.27)$$

$$\beta_{Q\Delta\theta_m.l} = \frac{\sigma_Q}{\sigma_{\Delta\theta_m}}\left(\frac{r_{Q\Delta\theta_m} - r_{Ql} r_{l\Delta\theta_m}}{1-(r_{l\Delta\theta_m})^2}\right) \quad (3.28)$$

3.5 Hypothesis Testing

For the purpose of this research, the following hypothesis are relevant to be tested

a. Null hypothesis (H_o): There is no significant relationship between heat transfer and length of heat exchanger with logarithmic mean temperature difference. H_o; $\beta_{Ql.\Delta\theta_m} = \beta_{Q\Delta\theta_m.l} = 0$
b. Alternative hypothesis (H_a): There is significant relationship between rate of heat transfer and length of exchanger with logarithmic mean temperature difference. H_a; $\beta_{Ql.\Delta\theta_m} \neq \beta_{Q\Delta\theta_m.l} = 0$

3.6 Model Fitting

Since this research utilized modified data from (Saad et, al) where they investigated the effect of length of heat exchanger on the rate of heat transfer for both parallel and counter flows but for the purpose of this work, the results of the counter flow shall be utilized due to reported accuracy over the parallel flow. This research considers the length of exchanger (l) and the logarithmic temperature difference ($\Delta\theta_m$) as two critical design parameters that influenced the rate of heat transfer (Q) and the results are tabulated in Table 5.1a. It is worthwhile to understand that only four (4) lengths of exchanger were originally considered to investigate the rate of heat transfer; this means only four data points were provided for analysis which are generally too small sample size for any statistical investigation. Hence, in order to generate more data points, linear extrapolation technique shall be used to provide as many data points/observations as possible, which in this case 20 data points (sample size) and in this case we shall be considering lengths from $(l = 5m, 10m, 15m, \ldots, 30m)$. The justification for the use of linear extrapolation technique is based on the following facts:

a. From literature review, it was revealed that the rate of heat transfer increased with increase in the length of the exchanger.
b. The lengths of exchanger used in their simulation varies linearly in steps of 5m starting from 10m till 25m.
c. The model to be fit in this research is a linear multiple regression model.

The points highlighted above is the backdrop of the idea to use the linear extrapolation technique to obtain the desired sample size for the analysis. Furthermore, it must be understood that the logarithmic temperature differences were not provided by [1] but only the inlet and outlet temperatures (boundary conditions) of the hot and cold fluids were provided together with the four lengths of the exchanger and their corresponding heat transfers values as shown in figure 3.2. Hence, we shall use necessary analytical methods to calculated the logarithmic mean temperature differences and use the observed pattern of the length of exchanger (l) to extrapolate corresponding data points for both the logarithmic temperature differences $(\Delta\theta_m)$ and the rate of heat transfer (Q).

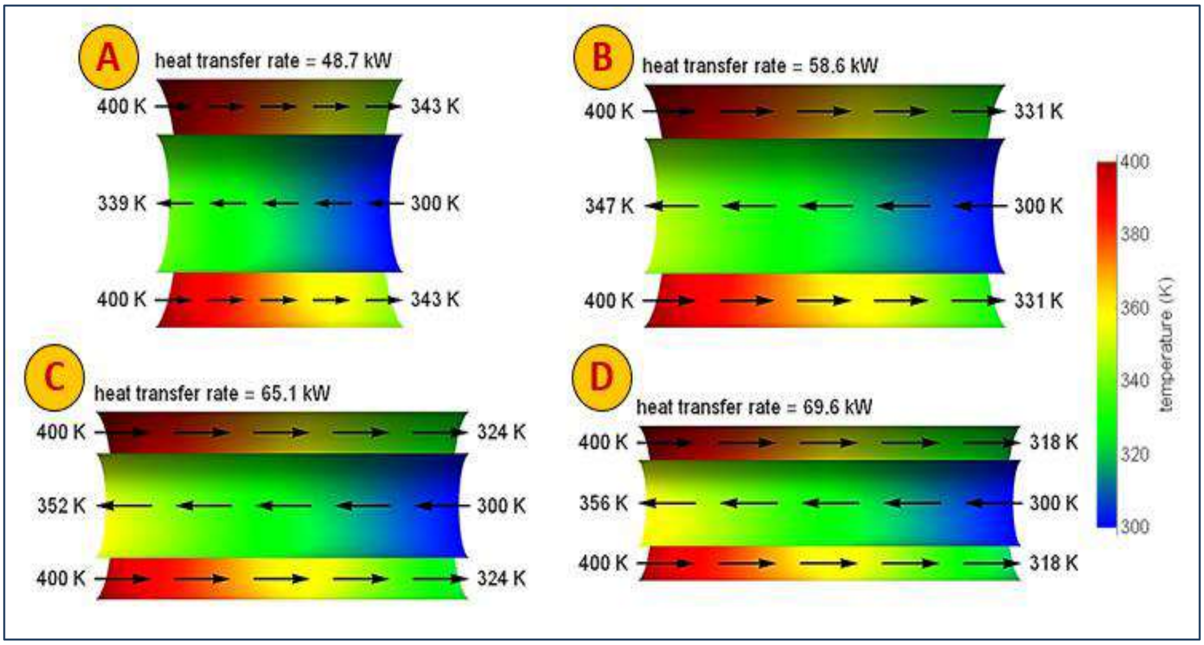

Figure 3.2 Results of counter flow heat exchanger for lengths: (A) 10m (B) 15m (C) 20m (D) 25m (Saad et al, 2022).
The inlet and outlet temperatures for both fluids can be easily seen figure 3.2 and are therefore tabulated in table 3.1a without additional computation.

3.8 The Logarithmic Mean Temperature Difference $(\Delta\theta_m)$

The logarithmic mean temperature difference across each length of the exchanger for the particular heat exchanger structural design is calculated as shown below with the assumptions that the heat exchanger is perfectly insulated from thermal interference of the sounding, constant specific heat of exchanger fluids, negligible axial conduction along the tube and constant overall heat transfer.

The exchanger temperature condition is given by:

$\Delta\theta_1 = \theta_{hi} - \theta_{co}$

$\Delta\theta_2 = \theta_{ho} - \theta_{ci}$

$$\Delta\theta_m = \frac{\Delta\theta_1 - \Delta\theta_2}{\ln(\Delta\theta_1/\Delta\theta_2)} \quad (5.29)$$

For l_{10}; $\Delta\theta_1 = \theta_{hi} - \theta_{co} = 400 - 339 = 61\text{K}$

$\Delta\theta_2 = \theta_{ho} - \theta_{ci} = 343 - 300 = 43\text{K}$

$\therefore$For l_{10}; $\Delta\theta_{m10} = \frac{\Delta\theta_1 - \Delta\theta_2}{\ln(\Delta\theta_1/\Delta\theta_2)} = \frac{61-43}{\ln(61/43)} = 51.5\text{K}$

For l_{15}; $\Delta\theta_1 = \theta_{hi} - \theta_{co} = 400 - 347 = 53\text{K}$

$\Delta\theta_2 = \theta_{ho} - \theta_{ci} = 331 - 300 = 31\text{K}$

$\therefore$For l_{15}; $\Delta\theta_{m15} = \frac{\Delta\theta_1 - \Delta\theta_2}{\ln(\Delta\theta_1/\Delta\theta_2)} = \frac{53-31}{\ln(53/31)} = 41\text{K}$

For l_{20}; $\Delta\theta_1 = \theta_{hi} - \theta_{co} = 400 - 352 = 48\text{K}$

$\Delta\theta_2 = \theta_{ho} - \theta_{ci} = 324 - 300 = 24\text{K}$

$\therefore$For l_{20}; $\Delta\theta_{m20} = \frac{\Delta\theta_1 - \Delta\theta_2}{\ln(\Delta\theta_1/\Delta\theta_2)} = \frac{48-24}{\ln(48/24)} = 34.6\text{K}$

For l_{25}; $\Delta\theta_1 = \theta_{hi} - \theta_{co} = 400 - 356 = 44\text{K}$

$\Delta\theta_2 = \theta_{ho} - \theta_{ci} = 318 - 300 = 18\text{K}$

$\therefore$For l_{25}; $\Delta\theta_{m25} = \frac{\Delta\theta_1 - \Delta\theta_2}{\ln(\Delta\theta_1/\Delta\theta_2)} = \frac{44-18}{\ln(44/18)} = 29\text{K}$

Table 3.1a Input and output counter flow heat exchanger parameters

S/n	Length (l)(m)	Inlet Temperatures Hot fluid (θ_{hi})(K)	Cold fluid (θ_{ci})(K)	Outlet Temperatures Hot fluid (θ_{ho})(K)	Cold fluid (θ_{co})(K)	Log mean Temp. Diff $(\Delta\theta_m)$ (K)	Heat Transfer (Q)(KW)
1	5	400	300	–	–	53	48
2	10	400	300	343	339	51.5	48.7
3	15	400	300	331	347	41	58.6
4	20	400	300	324	352	34.6	65.1
5	25	400	300	318	356	29	69.6
6	30	400	300	–	–	23	75.1
				–	–	–	

Source: Research Data (Saad et al, 2022)

From table 3.1a, some of the research data are missing for the inlet and outlet temperatures. Hence, the logarithmic temperature difference as well as the heat transfer are those that could not be directly deduced from the results of [1] in figure 3.2. Therefore, these points are extrapolated with the aid of the linear sequence observed in the lengths of the exchanger as follows:

Arbitrarily choosing the15m (l_{15}) as the datum; the values for the logarithmic temperature difference for greater lengths are extrapolated thus;

$$\Delta\theta_{m30} = \frac{l_{30}-l_{15}}{l_{25}-l_{15}}(\Delta\theta_{m25} - \Delta\theta_{m15}) + \Delta\theta_{m15} = \frac{30-15}{25-25}(29-41) + 41 = 23\text{K}$$

Similarly, the values for the heat transfer are also obtained as follows:

$$Q_{30} = \frac{l_{30}-l_{15}}{l_{25}-l_{15}}(Q_{m25} - Q_{m15}) + Q_{m15} = 75.1\text{KW}$$

Table 3.1b. Complete table of values for regression analysis

Length		Log mean Temp. Diff		Heat Transfer							
$(\boldsymbol{l})$(m)	dl	$(\Delta\theta_m)$	$d\Delta\theta_m$ (K)	(Q)(KW)	(dQ)	$dl\Delta d\theta_m$	$dQd\Delta\theta_m$	$dQdl$	dl^2	$d\Delta\theta_m{}^2$	dQ^2
5	-12.5	53	14.3	48	-12.9	-178.75	-184.47	161.25	156.25	204.49	166.41
10	-7.5	51.5	12.8	48.7	-12.2	-96	-156.16	91.5	56.25	163.84	148.84
15	-2.5	41	2.3	58.6	-2.3	-5.75	-5.29	5.75	6.25	5.29	5.29
20	2.5	34.6	-4.1	65.1	4.2	-10.25	-17.22	10.5	6.25	16.81	17.64
25	7.5	29	-9.7	69.6	8.7	-72.75	-84.39	65.25	56.25	94.09	75.69
30	12.5	23	-15.7	75.1	14.2	-196.25	-222.94	177.5	156.25	246.49	201.64
105		232.1		365.1		-559.75	-670.47	511.75	437.5 0	731.01	615.51

The mean of each of the variables is given by:

$$\bar{l} = \frac{\sum l}{n} = \frac{105}{6} = 17.5\ ;\ \overline{\Delta\theta_m} = \frac{\sum\Delta\theta_m}{n} = \frac{232.1}{6} = 38.7\ \bar{Q} = \frac{\sum Q}{n} = \frac{365.1}{6} = 60.9$$

$$\therefore \text{r}_{Ql} = \frac{\sum dQdl}{\sqrt{\sum dQ^2 \sum dl^2}} = \frac{511.75}{\sqrt{\sum(615.51)(437.5)}} = 0.9862$$

$$\text{r}_{Q\Delta\theta_m} = \frac{\sum dQd\Delta\theta_m}{\sqrt{\sum dQ^2 \sum d\Delta\theta_m{}^2}} = \frac{-670.47}{\sqrt{\sum(615.51)(731.01)}} = -0.9995$$

$$\text{r}_{l\Delta\theta_m} = \frac{\sum dld\Delta\theta_m}{\sqrt{\sum dl^2 \sum d\Delta\theta_m{}^2}} = \frac{-559.75}{\sqrt{\sum(437.50)(731.01)}} = -0.9898$$

$$\sigma_Q^2 = \frac{\sum dQ^2}{n} = \frac{615.51}{6} = 102.585 \Rightarrow \sigma_Q = 10.13;\ \sigma_l^2 = \frac{\sum dl^2}{n} = \frac{437.50}{9} = 72.92 \Rightarrow \sigma_l = 8.5;\ \sigma_{\Delta\theta_m}^2 = \frac{\sum d\Delta\theta_m{}^2}{n} = \frac{731.01}{6} = 121.835 \Rightarrow \sigma_{\Delta\theta_m} = 11.04$$

From; $\beta_{Ql.\Delta\theta_m} = \frac{\sigma_Q}{\sigma_l}\left(\frac{r_{Ql} - r_{Q\Delta\theta_m} r_{l\Delta\theta_m}}{1-(r_{l\Delta\theta_m})^2}\right) = \frac{10.13}{8.5}\left(\frac{0.9862 - (-0.9995)(-0.9898)}{1-(-0.9898)^2}\right) = \frac{10.13}{8.5}\left(\frac{-0.003105}{(0.020296)}\right) = -0.18$

Also, from (5.27); observe that $\beta_{Ql.\Delta\theta_m}$ is a dimensionless quantity since it is a mere coefficient that only measures the rate of change of the heat transfer with respect to change in the length of exchanger.

$$\beta_{Q\Delta\theta_m.l} = \frac{\sigma_Q}{\sigma_{\Delta\theta_m}}\left(\frac{r_{Q\Delta\theta_m} - r_{Ql} r_{l\Delta\theta_m}}{1-(r_{l\Delta\theta_m})^2}\right) = \frac{10.13}{11.0}\left(\frac{(-0.9995)-(0.9862)(-0.9898)}{1-(-0.9898)^2}\right) = \frac{10.13}{11.04}\left(\frac{-0.023359}{(0.020296)}\right) = -1.06$$

OR

$$dQ = \frac{\sigma_Q}{\sigma_l}\left(\frac{r_{Ql} - r_{Q\Delta\theta_m} r_{l\Delta\theta_m}}{1 - \left(r_{l\Delta\theta_m}\right)^2}\right) dl + \frac{\sigma_Q}{\sigma_{\Delta\theta_m}}\left(\frac{r_{Q\Delta\theta_m} - r_{Ql} r_{l\Delta\theta_m}}{1 - \left(r_{l\Delta\theta_m}\right)^2}\right) d\Delta\theta_m \qquad (3.30)$$

Equation (5.30) is the required model for the Heat Transfer with respect to thermo – spatial properties; Length of Exchanger and Logarithmic Mean Temperature Difference (LMTD) When The Observations Are Measured from their respective means.
However, if the observations are not measured from their respective means but the origin, then the equivalent form of the heat transfer model is given according to (5.32).

$$Q = Q_m + \frac{\sigma_Q}{\sigma_l}\left(\frac{r_{Ql} - r_{Q\Delta\theta_m} r_{l\Delta\theta_m}}{1 - \left(r_{l\Delta\theta_m}\right)^2}\right) dl + \frac{\sigma_Q}{\sigma_{\Delta\theta_m}}\left(\frac{r_{Q\Delta\theta_m} - r_{Ql} r_{l\Delta\theta_m}}{1 - \left(r_{l\Delta\theta_m}\right)^2}\right) d\Delta\theta_m \qquad (3.31)$$

Where $Q_m = \bar{Q}$ is the mean heat transfer of the investigated system. Care must be taken to understand that Q_m is not same as the constant of the model (β_o) which basically implies the minimum amount of heat transfer required in the heat exchanger system.
Equation (5.30) is the required model for the Heat Transfer with respect to the Length of Exchanger and Logarithmic Temperature Difference

3.7 Coefficient of Multiple Determination ($R^2_{Q.l\Delta\theta_m}$)

To test the validity and strength of the model, we evaluate the coefficient of multiple determination ($R^2_{Q.l\Delta\theta_m}$) which explains the realistic amount of linear relationship between the heat transfer, length of exchanger and the logarithmic mean temperature difference according to the model. It is expressed according Yule's notation as:

$$R^2_{Q.l\Delta\theta_m} = \frac{r^2_{Ql} + r^2_{Q\Delta\theta_m} - 2 r_{Ql} r_{Q\Delta\theta_m} r_{l\Delta\theta_m}}{1 - r^2_{l\Delta\theta_m}} \qquad (3.32)$$

$$\Rightarrow R^2_{Q.l\Delta\theta_m} = \frac{(0.9862)^2 + (-0.9995)^2 - 2[(0.9862)(-0.9995)(-0.9898)}{1 - (-0.9817)^2} = \frac{0.020285}{0.020296} = 0.9995 \ \textbf{\textit{ans.}}$$

3.8 F-test For The Model

The Fisher's test otherwise known as the F-test is generally used for establishing statistical grounds upon which relevant inferences can be drawn about the nature and strength of relationship between the explanatory and response variables for the regression model. However, the nitty-gritty of the processes involved in establishing the basis for which relevant inferences are made are uncovered in the Analysis of Variane (ANOVA) of the model. The ANOVA process involves the structural outlook of a table which summarizes the breakdown of the variability in the response variable. The variability takes the following form.

$$\text{F} = \frac{\text{Variation due to model}}{\text{Variation due to error}} = \frac{\text{Model mean square}}{\text{Error mean square}} \qquad (3.32)$$

Table 3.2 Analysis of variance (anova) for the regression model

Length	Log mean Temp. Diff	Heat Transfer	Total Treatment Total Sum of Squares (SSE_T)			Within Treatments Sum of Squares (SSE_w)		
(l)(m)	($\Delta\theta_m$)	(Q)(KW)	dl^2	$d\Delta\theta_m{}^2$	dQ^2	dl^2	$d\Delta\theta_m{}^2$	dQ^2
5	53	48	1156.68	195.72	80.82	156.25	204.49	166.41
10	51.5	48.7	841.58	156.00	93.90	56.25	163.84	148.84
15	41	58.6	576.48	3.96	383.77	6.25	5.29	5.29
20	34.6	65.1	361.38	19.45	680.69	6.25	16.81	17.64
25	29	69.6	196.28	100.20	935.75	56.25	94.09	75.69
30	23	75.1	81.18	256.32	1302.49	156.25	246.49	201.64
105	232.1	365.1	3213.58	731.65	3477.42	437.50	731.01	615.51

Grand mean; $\bar{\text{G}} = \frac{\sum l + \sum \Delta\theta_m + \sum Q}{N} = \frac{105 + 232.1 + 365.1}{18} = 39.01$

Total Treatment; $\text{SSE}_T = \sum(\text{G}_i - \bar{\text{G}})^2 = \sum(l_i - \bar{\text{G}})^2 + \sum\left(\Delta\theta_{m\,i} - \bar{G}\right)^2 + \sum(Q_i - \bar{\text{G}})^2$

$\Rightarrow \text{SSE}_T = \sum(\text{G}_i - \bar{\text{G}})^2 = 3213.58 + 731.58 + 3477.42 = 7422.16$

Within Treatments; $\text{SSE}_w = \sum\left(l_i - \bar{l}\right)^2 + \sum\left(\Delta\theta_{m\,i} - \overline{\Delta\theta_m}\right)^2 + \sum(Q_i - \bar{Q})^2$

$\Rightarrow \text{SSE}_w = 437.50 + 731.65 + 615.51 = 1784.66$

Between Treatments; $\text{SSE}_b = \text{SSE}_T - \text{SSE}_w = 7422.16 - 1784.66 = 5637.5$

Degrees of freedom between treatments/regression model; $= k = 2$

Degrees of freedom within treatments/residuals; $= n - k - 1 = 18 - 2 - 1 = 15$

Where k is the number of explanatory variables in the model and n the total number of observations in the data.
The F-test as given by (3.32) can as well be summarized in as shown in Table 3.3

Table 3.3 F-test summary for the model

	df	SS	MS	F
Regression	$k = 2$	$SSE_b = 5637.5$	$MSR = SSE_b/k$ $MSR = 5637.5/2$ $= 2818.75$	$F = MSR/MSE$ $= 2818.75/118.75$ $= 23.69$
Residual	$n - k - 1 = 15$	$SSE_w = 1784.66$	$MSE = SSE_w/df$ $MSE = 1784.66/15$ $= 118.98$	
Total	$n - 1 = 17$	$SSE_T = 7422.16$		

@ $\alpha = 0.05; F_{critical} = 3.68$

Since the F-value from the model is greater than the critical value i.e. $F_{cal} = 23.69 > 3.68$; we therefore reject the null hypothesis and accept the alternative hypothesis for the model.

4. Results and Discussion

The model revealed a minimum heat transfer of zero (0kw) at zero length and logarithmic mean temperature difference measured from their respective means which is intuitively correct and supported by the fundamental assumption of Yule's method that the multiple regression constant is zero ($\beta_o = 0$) at these boundary conditions of ($dl = 0$ and $\sigma_{\Delta\theta_m} = 0$) since the variables are measured from their respective means. This implies the model does not support negative heat transfer values but only positive value, which is theoretically and practically correct and in line with the findings of [1 – 5]. However, the model reveal that the logarithmic mean temperature difference cannot be extrapolated to zero ($\sigma_{\Delta\theta_m} = 0$) at zero length of exchanger ($l = 0$) which is in line with provisions of the third law of thermodynamics which states that it is impossible for any system to reach absolute zero with finite number of processes. Which in turn gives a corollary that efficiency of the exchanger cannot be 100% due to heat losses.

This is further true because if the logarithmic temperature difference had reached zero, the hot and cold fluid would be in thermal equilibrium and there would be zero heat transfer between them as provided by the Zeroth Law of thermodynamics. Hence it can be said that the highest temperature of the cold fluid never rose to the lowest temperature of the hot fluid hence, the system never attained thermal equilibrium in accordance with investigations of [1], [2] and [5]. However, model indicates that heat transfer increases with length of the exchanger and logarithmic temperature difference as also suggested by [1]. The model also revealed a good fit for the data due to symmetrical extrapolation of data points and residuals show minimal errors in line with the least square regression principle.

The coefficient of multiple determination suggests that most of the observed variations in the heat transfer are due to the length of exchanger and logarithmic temperature difference while only small portion of the variations in the heat transfer are due to extraneous factors.

The F-statistics also invalidated the null hypothesis and accepted the alternative hypothesis $\alpha = 0.05$ significance since the $F_{cal} > F_{critical}$ from the analysis of variance of the sources of errors in the data.

5. Conclusion and Recommendations

Overall, it is evident from the results that the Yule's multiple regression model is effective for deriving a deterministic model for the thermo – spatial properties of an HE with respect to rate of heat transfer, length of exchanger and logarithmic temperature difference between the hot and cold fluids across the exchanger and indeed with respect to any number of explanatory variables of interest provided the data has been properly collected with symmetry and the assumptions of multiple regression analysis are upheld. Length of exchanger and logarithmic temperature difference have very significant effect on the amount of heat transfer for the particular type heat exchanger considered in this study. However, it is recommended that:

a. more explanatory variables and greater sample size should be considered for broader investigation and understanding of the exchanger system performance
b. the analysis should be applied to superheated heat exchanger systems for wider temperature analysis where the regression model can be used to extrapolated to zero logarithmic temperature difference
c. Exchanging fluid with better physical properties should be considered as replacement for the water medium used in the is analysis
d. More complex heat exchanger designs should be considered to derive mathematical models for the critical design variables to understand performance and optimization of particular type HE
e. Robust residual analysis should be carried out to further validated the model
f. Artificial intelligence models such as logistic regression should be considered.

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