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Mathematical Model of Youth Gambling Dynamics in Nigeria with Incorporation of Digital Exposure, Financial Debt, and Policy Enforcement

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ABSTRACT

In recent years, game betting among the youth has become a significant social and public health concern not only in developing nations like Nigeria but also in developed nations, due to rapid global digitalization, aggressive online betting promotions, availability of modern internet devices, and increasing financial vulnerability. In this investigation, a designed deterministic compartmental mathematical model was utilized to survey the dynamic forces behind youth gambling by explicitly incorporating digital and internet exposure, gambling addiction progression, financial debt accumulation, relapse vulnerability, and policy enforcement mechanisms. The youths' population was stratified into susceptible S(t), exposed E(t), betting B(t), problem gambling P(t), recovering R(t), vulnerable V(t), and debt-burdened compartments D(t). Using the next-generation matrix method, the effective reproduction number parameters were obtained as $\beta=0.4yr^{-1}$, $\alpha=0.2yr^{-1}$, $\gamma=0.15yr^{-1,}$ and θ=0.25yr-1, while the gambling-free equilibrium and the endemic gambling equilibrium were derived equationally. Local and global stability analyses revealed how the gambling-free equilibrium was stable when the effective reproduction number was less than unity. While the global stability of the endemic gambling equilibrium was derived, the normalized forward sensitivity identified digital and peer influence (β=+1.00) as the most dominant factors enhancing gambling transmission, rehabilitation (θ=-0.31), and addiction progression rates (α=-0.29), whereas natural demographic exit (μ) showed minimal impact. The numerical simulations supporting analytical results validate financial debt and relapse (ε) mechanisms significantly sustaining gambling prevalence. These findings demonstrate that integrated policy interventions combining advertising regulation, accessible rehabilitation services, debt mitigation strategies, and effective enforcement can achieve sustainable reductions in youth gambling in Nigeria.

1. Introduction

Games betting a form of gambling, particularly online sports betting among the youth, has witnessed rapid growth in Nigeria over the past decade, largely driven by increased internet penetration, mobile technology, and aggressive digital advertising targeting youths. While betting companies are often promoted as sources of entertainment and employment, mounting evidence indicates that excessive gambling among youths is associated with financial distress, academic underperformance, mental health challenges, and social instability [1, 2]. In Nigeria, for instance, weak regulatory enforcement and easy access to online betting platforms have further intensified youth participation in gambling activities [3]. Currently, the utilization of mathematical modelling approaches seems to provide a powerful background for understanding the transmission and persistence of gambling behaviour in populations, analogous to the spread of social behaviours and addictions. However, unfortunately, the existing gambling models have largely focused on initiation and addiction dynamics, with limited consideration of digital exposure, relapse mechanisms, and financial debt, which are critical drivers of gambling persistence behaviour, especially in developing economies [4, 5]

Additionally, the role of debt in reinforcing gambling behaviour and undermining recovery efforts remains underexplored in the Nigerian context. Meanwhile, so far, the existing mathematical models of gambling behaviour, including CBT-based models and the reports of [6], share common features such as peer-influenced initiation, progression to a gambling problem, recovery, and relapse dynamics.

However, from the foregoing, it is important to establish here that critical Nigeria-specific and modern drivers of gambling are not explicitly captured by most authors in the literature, such as digital/algorithmic exposure (mobile apps, social media ads, bonus notifications), debt accumulation and financial distress (which independently worsen addiction), regulatory and policy enforcement effects (advert bans, betting limits, age verification), economic shocks (unemployment, inflation spikes), and Household spillover effects (family financial stress feeding back into relapse). Nevertheless, to address these gaps, we extend our model by introducing three new state variables and time-dependent control mechanisms, while keeping the model analytically tractable. The newly introduced compartment is first, digitally exposed class E(t), which represents youths who are yet to be involved in betting, but are repeatedly exposed to online betting ads, social media influencers, promotional bonuses, and free bets. Such exposure encourages gambling behaviour, even without direct peer contact. Notably, the current gambling models largely have failed to account for this mechanism, which suggests that initiation occurs only through contact with active gamblers. Secondly, financially distressed class D(t), which represents gamblers (or former gamblers) trapped in debt, loan defaults, or informal borrowing, which accelerates progression to problem gambling P(t), increases relapse probability, and reduces recovery success.

In Nigeria, debt acts as a structural driver, fuelled by youth unemployment and reliance on informal lending. Thirdly, policy-control state C(t), which is the control variable and represents the strength of gambling regulation and enforcement, including advertising bans, betting stake limits, app restrictions, and campus betting prohibitions. This approach differs from most models by incorporating policy measures that dynamically suppress transmission parameters. Additionally, the new transmission terms and controls such as digital exposure force of influence ($a1 = (1 - C)\omega$) and $C \in (0,1)$ is the regulatory enforcement level, transition from exposure to betting ($a2 = 1 - C)\tau$ capturing the influence of algorithmic nudges (push notifications, and bonuses) and lastly, debt-driven escalation (κ) which is the rate at which debt worsens problem gambling P(t) and (η) which is debt-induced relapse rate from vulnerability V(t). These feedback mechanisms have been overlooked in most existing models. The aim of this study therefore is to incorporate the last identified parameters lacking in other models of studies reported elsewhere in the literature by different authors and researchers in the present study to adequately assess and analyse extensively the role of the various variables in promoting and sustaining games' batting also known as gambling in Nigeria and the consequences of habitual gambling behaviour particularly among the youth.

2. Materials and Methods

Here, the mathematical model of youth gambling dynamics in Nigeria with incorporated digital exposure, financial debt, and policy enforcement has been developed and analysed. To establish the workability of the designed model, the mathematical properties of the model, such as positivity of the gambling model and invariant region of the gambling model, were equally analysed in order to find out that the model is well modelled mathematically and meaningfully socially.

The Gambling Model

In the designed model, the individuals that form the susceptible class S(t) by recruitment are described at the rate of (Λ), and the susceptible individual, S(t), can become exposed E(t) to gambling at the rate of (a_1S). Furthermore, the natural exit from the population is considered to occur at the rate of (μS), while the exposed individuals progress to active betting B(t) at the rate of (a_2), and the natural exit occurs at the rate of (μE). Moreover, additional recruitment into betting is considered to occur through peer and digital influence at the rate of $\beta S\frac{(B+P)}{N}$ reflecting social reinforcement, whereas bettors progress to problem gambling P(t) at the rate of (αB). Yet again, while some bettors quit gambling and then join the vulnerable group, V(t) is considered to occur at the rate of (γB), the natural exit could further occur at the rate of (μB). Similarly, while the individuals who relapse into problem gambling state P(t) from the vulnerable group V(t) is considered to occur at the rate of (δV), the debt-burdened individuals re-entering problem gambling P(t) occur at the rate of (κD). Meanwhile, as the problem gamblers P(t) is considered to enter the recovery state through rehabilitation R(t) at the rate of (θP), the natural exit is also considered to occur at the rate of (μP).

Once more, the entry occurring from the problem gambling class P(t) is considered to occur at the rate of (θP). In contrast, some recovering individuals who relapse into vulnerability class V(t) are considered to occur at the rate of (σR). Their natural exit is well thought-out to occur at the rate of (μR). Nonetheless, while entry occurring from bettors who have quit gambling is considered to occur at the rate of (γB), their recovering individuals who become vulnerable after treatment may occur at the rate of (σR), and their vulnerable individuals who relapse into problem gambling are equally well thought-out to occur at the rate of (δV). Remarkably, financial stress causing vulnerable individuals to get into serious debt is considered to occur at the rate of (ηDV), whereas their natural exit is considered to occur at the rate of (μV). In other considerations, while the entry that may occur from both betting B(t) and problem gambling (P) classes is considered to occur at the rate of $\rho(B+P)$, the debt-burdened individuals re-entering problem gambling is considered to occur at the rate of (κD), and the resolution of debt or financial intervention is considered to occur at the rate of (ϕD), whereas their natural exit may occur at the rate considered as (μD).

The Assumptions of the Gambling Model

(i) Are youths' uniform interaction and exposure to gambling through peers and digital platforms occurring at an average rate?

(ii) Is entry into gambling primarily influenced by online advertisements, social media, and mobile betting platforms?
(iii) Are youths who accumulate financial debt more likely to continue with gambling and less likely to recover spontaneously?
(iv) Can regulatory measures such as self-exclusion and advertising restrictions reduce gambling initiation and increase exit from gambling behaviour?

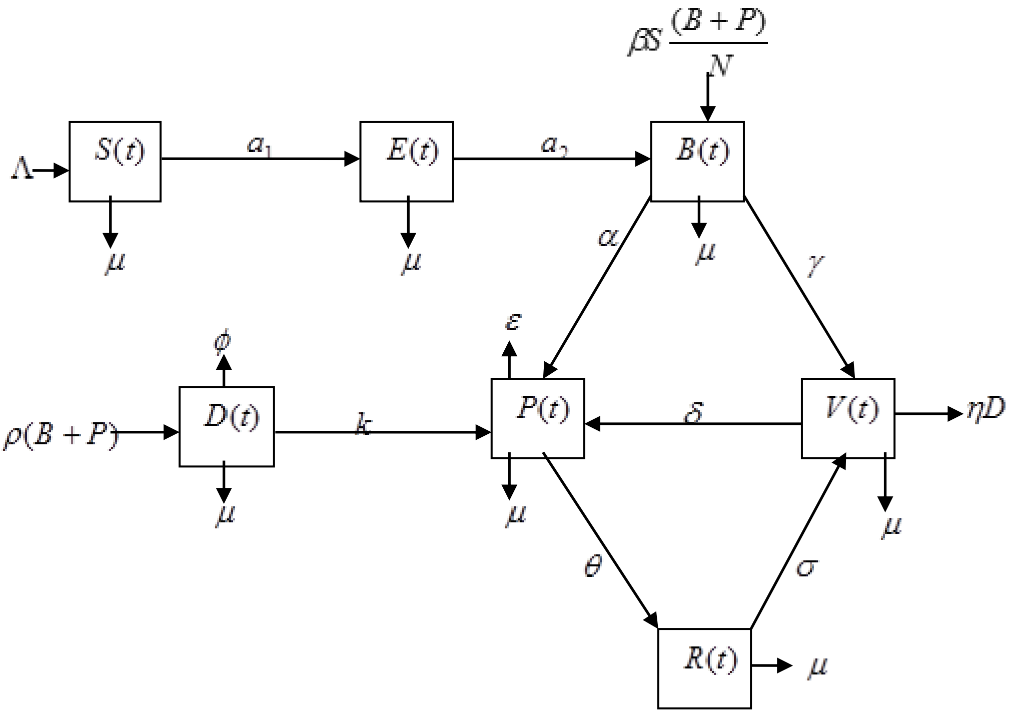

Figure 1: The Flow Diagram of the Designed Gambling Model.

The Equations Derived from the Designed Gambling Model

From the designed gambling model, important equations presented below were obtained considering the modelled flow diagram:

$$\frac{dS}{dt} = \Lambda - a_1 S - \mu S, \quad \ldots(1)$$

$$\frac{dE}{dt} = a_1 S - a_2 E - \mu E, \quad \ldots(2)$$

$$\frac{dB}{dt} = a_2 E + \frac{\beta S(B+P)}{N} - (\alpha+\gamma+\mu)B, \quad \ldots(3)$$

$$\frac{dP}{dt} = \alpha B + \delta V + kD - (\theta+\varepsilon+\mu)P, \quad \ldots(4)$$

$$\frac{dR}{dt} = \theta P - (\sigma+\mu)R, \quad \ldots(5)$$

$$\frac{dV}{dt} = \gamma B + \sigma R - (\delta+\eta D+\mu)V, \quad \ldots(6)$$

$$\frac{dD}{dt} = \rho(B+P) - (k+\phi+\mu)D, \quad \ldots(7a)$$

Table 1: State Variables of the Gambling Model

State Variables	Definition/Interpretation of Each Variable
$S(t)$	Youths who are not in involved in gambling but are potentially exposed through social interaction, media, or environmental factors. These individuals have not yet developed gambling behaviour.
$E(t)$	youths who are not yet betting but are repeatedly exposed to online betting advertisements, mobile apps, promotions, and influencer marketing. This class captures pre-initiation digital conditioning.
$B(t)$	youths who engage in occasional or recreational betting, such as sports betting, without severe addiction symptoms. Individuals in this class may either stop gambling, escalate to problem gambling, or remain recreational bettors.

$P(t)$	youths suffering from gambling addiction, characterized by compulsive betting behaviour, financial loss, emotional distress, and social dysfunction. This class is central to the public health burden of gambling.
$R(t)$	youths who are undergoing recovery or rehabilitation, including counseling, therapy, self-exclusion programs, or informal family support.
$V(t)$	former gamblers who have temporarily stopped betting but remain highly susceptible to relapse, particularly under economic pressure or social exposure.
$D(t)$	youths experiencing gambling-related financial distress, such as debt accumulation, loan defaults, or informal borrowing.

Table 2: Parameters of the Gambling Model

Model Parameter	Definition/Interpretation of Each Model Parameter
Λ	Recruitment rate into the youth population.
μ	Natural exit rate (aging out, migration, or death).
w	Intensity of digital betting advertisements.
τ	The rate at which digital exposure leads to betting initiation.
β	Peer-influence betting initiation rate.
$a_1 = (1-C)w$	Transition rate from S to E
$a_2 = (1-C)\tau$	The rate of transition from E to B
α	The rate of progression from recreational betting to problem gambling.
γ	The rate at which bettors stop gambling and become vulnerable.
δ	Relapse rate from vulnerability to problem gambling.
σ	The rate at which the recovering persons turn out to be vulnerable
k	The rate in which poor financial status/distress worsens problem gambling
η	Debt-induced relapse amplification rate
θ	Recovery rate of problem gamblers
ε	Gambling-induced severe exit (e.g., incarceration, extreme distress)
ϕ	Debt resolution or financial exit rate
$C \in [0,1]$	Level of policy enforcement and regulation
ρ	Rate of debt accumulation among bettors and problem gamblers

Analysis of Positivity of the Gambling Model

Considering the equations of the model, (1) to (7), it could be assumed to be positive if, for any initial conditions; $\{S(0), E(0), B(0), P(0), R(0), V(0), D(0)\} \in R_+^7$, the solution satisfies $\{S(t), E(t), B(t), P(t), R(t), V(t), D(t)\} \in R_+^7$ for all $t > 0$.

Proof: We have to prove the positivity of each compartment by showing that the solutions cannot cross the coordinate hyperplanes. From equation (1);

$$\frac{dS}{dt} = \Lambda - (a_1 + \mu)S\,; \quad \text{(7b)}$$

Solving by applying the method of integration, the linear ordinary differential equation (ODE) solution becomes;

$$S(t) = S(0)e^{-(a_1+\mu)t} + \frac{\Lambda}{a_1+\mu}(1 - e^{-(a_1+\mu)t})\,; \quad \text{(8a)}$$

Since $\Lambda \geq 0$, $a_1 \geq 0$, and $S(0) \geq 0$. Therefore, $S(t) \geq 0$ for all $t > 0$.

Then, from equation (2);

$$\frac{dE}{dt} = a_1 S - (a_2 + \mu)E\,; \quad \text{(8b)}$$

At E = 0:

Thus; $$\frac{dE}{dt} = a_1 S \geq 0\,; \quad \text{(9)}$$

Hence, the trajectories cannot cross into E < 0. Therefore, $E(t) \geq 0$ for all t > 0.

From equation (3);

$$\frac{dB}{dt} = a_2 E + \frac{\beta S(B+P)}{N} - (\alpha + \gamma + \mu)B\,.$$

At B = 0: hence;

$$\frac{dB}{dt} = a_2 E + \frac{\beta SP}{N} \geq 0.; \quad \text{(10a)}$$

Since all state variables are non-negative. Hence, $B(t) \geq 0$ for all t > 0.

From equation (4);

$$\frac{dP}{dt} = \alpha B + \delta V + kD - (\theta + \varepsilon + \mu)P; \quad \text{(10b)}$$

At P = 0: hereafter;

$$\frac{dP}{dt} = \alpha B + \delta V + kD \geq 0; \quad \text{(11a)}$$

Thus, as the problem gamblers remain non-negative, therefore, $P(t) \geq 0$ for all t > 0.

From equation (5);

$$\frac{dR}{dt} = \theta P - (\sigma + \mu)R; \quad \text{(11b)}$$

At R = 0: therefore;

$$\frac{dR}{dt} = \theta P \geq 0; \quad \text{(12a)}$$

Therefore, $R(t) \geq 0$ for all t > 0.

From equation (6);

$$\frac{dV}{dt} = \gamma B + \sigma R - (\delta + \eta D + \mu)V; \quad \text{(12b)}$$

At V = 0: then;

$$\frac{dV}{dt} = \gamma B + \sigma R \geq 0; \quad \text{(13a)}$$

Hence, $V(t) \geq 0$ for all t > 0.

From considering equation (7);

$$\frac{dD}{dt} = \rho(B + P) - (k + \phi + \mu)D; \quad \text{(13b)}$$

At D = 0: thus;

$$\frac{dD}{dt} = \rho(B + P) \geq 0; \quad \text{(14)}$$

Thus, as $D(t) \geq 0$ for all t > 0.

Therefore, since all the equations (8) to (14) have shown that for any non-negative initial conditions, all the state variables of the gambling model remain non-negative for all future time. Hence, the model is mathematically well modelled and socially meaningful.

The Invariant Region of the Gambling Model.

To ascertain the boundedness of the population, we examine the growth of the total population N(t). Summing all the equations (1) to (7) results in;

$$\frac{dN}{dt} = \Lambda - \mu(S + E + B + P + R + V + D) - \varepsilon P - \phi D - \eta DV; \quad \text{(15)}$$

Since all the state variables are non-negative in (15), we have;

$$\frac{dN}{dt} = \Lambda - \mu N; \quad \text{(16)}$$

Then, by solving (16) by variable separable and integrating both sides with the initial condition;

N(t) = N(0), we have;

$$N(t) = \frac{\Lambda}{\mu} + (N(0) - \frac{\Lambda}{\mu})e^{-\mu t}; \quad \text{(17)}$$

By comparison theorem;

$$0 \leq N(t) \leq \frac{\Lambda}{\mu}; \text{ where all } t \geq 0.$$

Thus, $\limsup_{t\to\infty} N(t) \le \frac{\Lambda}{\mu}$.

To evaluate further, we formulate Theorem 1:

For any initial condition in $\Omega = \left\{ (S,E,B,P,R,V,D) \in R_+^7 : N \le \frac{\Lambda}{\mu} \right\}$, the solution of the gambling model exists for all $t \ge 0$ and remains in Ω.

Proof:

On the boundary $N = \frac{\Lambda}{\mu}$. This implies that, $\frac{dN}{dt} = \Lambda - \mu(\frac{\Lambda}{\mu}) = 0$.

Therefore, the vector field points inward or tangential to the boundary, and trajectories cannot cross outward. Also, since we have already established that each coordinate hyperplane (S =0, E = 0, B = 0, P = 0, R = 0, V = 0, D = 0) is inward pointing. Hence, no solution leaves the non-negative orthant. Also, since all compartments are non-negative and their sum is bounded above by, $\frac{\Lambda}{\mu}$, each compartment satisfies: $0 \le X(t) \le \frac{\Lambda}{\mu}, X \in \{S,E,B,P,R,V,D\}$. Thus, every state variable is uniformly bounded.

Conclusively, the gambling model admits a socially feasible invariant region in which all solutions are non-negative and bounded. This guarantees that the model is well-posed and provides a solid foundation for equilibrium and stability analyses.

Gambling-Free Equilibrium of the Extended Model

The gambling-free equilibrium (GFE) refers to the situation in which no individual is involved in gambling or its consequences. Hence, all gambling-related compartments are set to zero. That is,

B = P = R = V = D = 0; ……………………………………………………………………….(18a)

At equilibrium, all the time derivatives vanish, and by substituting equation (18) into equations (1) to (7), we have;

$$\Lambda - (a_1 + \mu)S = 0; \quad \text{.....(18b)}$$

$$a_1 S - (a_2 + \mu)E = 0; \quad \text{..... (18c)}$$

Then, in solving for S_0 in equation (18b), we obtain;

$$S_0 = \frac{\Lambda}{a_1 + \mu}; \quad \text{..... (19)}$$

Also, solving for E_0 in equation (18c), it results to;

$$E_0 = \frac{a_1 S_0}{a_2 + \mu} = \frac{a_1 \Lambda}{(a_1 + \mu)(a_2 + \mu)}; \quad \text{.....(20a)}$$

Consistency of the gambling-free condition

For the gambling-free equilibrium to hold, we must set:

$$\frac{dB}{dt} = a_2 E = 0.$$

This implies that $a_2 = 0$ or $E = 0$. Since $a_2 = (1-C)\tau$, it's either $C = 1$ or $\tau = 0$.

Hence, the digital exposure alone cannot persist without generating gamblers unless conversion is completely blocked. Therefore, under effective control ($a_2 = 0$), the unique gambling-free equilibrium, G_0 is given by;

$$G_0 = \left[\frac{\Lambda}{(a_1 + \mu)}, \frac{a_1 \Lambda}{(a_1 + \mu)(a_2 + \mu)}, 0,0,0,0,0 \right]; \quad \text{.....(20b)}$$

Additionally, if $a_1 = 0$ (total advertising ban), then

$$G_0 = \left[\frac{\Lambda}{\mu}, 0,0,0,0,0,0 \right]; \quad \text{..... (20c)}$$

Effective Reproduction Number of the Gambling Model

The effective reproduction number, symbolized by Re, denotes the average number of new gamblers produced by an individual gambler during their gambling lifetime in a population where control measures (policy enforcement, regulation) are in place. By utilizing the next-generation methodological approach developed by Driessche and Watmough [7], it is assumed that gambling transmission occurs through the following gambling compartments:

$$X = (E, B, P)^T \; ; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(20d)}$$

Decomposition into new-gambling and transition terms, we have;

$$\frac{dX}{dt} = F(X) - V(X) \; ; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(20e)}$$

where 'F' is the rate of new gambling generation and 'V' is the transfers and removals rate.

Therefore, the new gambling terms; $$F = \begin{pmatrix} (1-C)wS \\ a_1E + \dfrac{\beta S(B+P)}{N} \\ 0 \end{pmatrix} \; ; \ldots\ldots\ldots\ldots\ldots\ldots \text{(20f)}$$

The transition terms $$V = \begin{pmatrix} (a_1+\mu)E \\ (\alpha+\gamma+\mu)B \\ (\theta+\varepsilon+\mu)P - \alpha B \end{pmatrix} ; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(20g)}$$

Jacobian matrices at the gambling-free equilibrium, G_0, will be;

Jacobian of $$F = \begin{pmatrix} 0 & 0 & 0 \\ a_1 & \beta\dfrac{S_0}{N} & \beta\dfrac{S_0}{N} \\ 0 & 0 & 0 \end{pmatrix} ; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(21)}$$

Where, $N = S_0 + E_0$ and then;

$$V = \begin{pmatrix} a_1+\mu & 0 & 0 \\ 0 & \alpha+\gamma+\mu & 0 \\ 0 & -\alpha & \theta+\varepsilon+\mu \end{pmatrix} ; \ldots\ldots\ldots\ldots\ldots\text{(21b)}$$

Hence, by applying the inverse of the matrix 'V', Equation 22 was obtained:

$$V^{-1} = \begin{pmatrix} \dfrac{1}{a_1+\mu} & 0 & 0 \\ 0 & \dfrac{1}{\alpha+\gamma+\mu} & 0 \\ 0 & \dfrac{\alpha}{(\alpha+\gamma+\mu)(\theta+\varepsilon+\mu)} & \dfrac{1}{\theta+\varepsilon+\mu} \end{pmatrix} ; \ldots\ldots\ldots\text{(22a)}$$

However, the next generation matrix given as ($K = FV^{-1}$) was determined by multiplying (21) and (22a), thus;

$$K = \begin{pmatrix} 0 & 0 & 0 \\ \dfrac{a_2}{a_2+\mu} & \dfrac{\beta S_0}{(\alpha+\gamma+\mu)N} & \dfrac{\beta S_0}{(\theta+\varepsilon+\mu)N} \\ 0 & 0 & 0 \end{pmatrix} ; \ldots\ldots\ldots\ldots\ldots \text{(22b)}$$

The eigenvalues of K are found by considering that;

$$\lambda_1 = 0,\; \lambda_2 = 0,\; \lambda_3 = \frac{\beta S_0}{(\alpha+\gamma+\mu)N} + \frac{\beta S_0}{(\theta+\varepsilon+\mu)N} ; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(22c)}$$

Hence, the R_e, which is the largest eigenvalue of K, will be achieved as;

$$R_e = \frac{\beta S_0}{N}\left(\frac{1}{(\alpha+\gamma+\mu)} + \frac{1}{(\theta+\varepsilon+\mu)}\right) ; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(23)}$$

Substituting $S_0 = \dfrac{\Lambda}{a_1+\mu}$ and $N = \dfrac{\Lambda}{\mu}$ into (23), thus;

$$R_e = \frac{\beta\mu}{a_1+\mu}\left(\frac{1}{(\alpha+\gamma+\mu)} + \frac{1}{(\theta+\varepsilon+\mu)}\right) ; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(24)}$$

The Stability Threshold

Theorem 2: It is believed that the gambling-free equilibrium of the gambling model is *locally asymptotically stable* if $R_e < 1$ and *unstable* if $R_e > 1$.
Proof:
The effective reproduction number (24) derived generalizes standard gambling models by incorporating digital exposure and policy control. It provides a quantitative threshold linking advertising, peer influence, and recovery mechanisms, offering a powerful tool for evaluating gambling interventions in Nigeria. The effective reproduction number was obtained by employing the breakdown method reported by [7]. Thus, a basic reproduction number determined through this method describes the local stability of the gambling-free equilibrium state, which is locally asymptotically stable when $R_e < 1$, and unstable when $R_e > 1$.

Endemic Gambling Equilibrium State of the Gambling Model
An endemic gambling equilibrium (EGE) is a steady state at which gambling persists permanently in the population. At this equilibrium, at least one gambling-related compartment is strictly positive. That is, $B^* > 0$ and $P^* > 0$.
To determine the endemic equilibrium, we considered the endemic equilibrium to be denoted by $E_1 = (S^*, E^*, B^*, P^*, R^*, V^*, D^*)$;
At equilibrium, let all the derivatives of Equations (1) to (7) be set to zero to give;

$$0 = \Lambda - a_1 S - \mu S; \quad (25)$$

$$0 = a_1 S - a_2 E - \mu E; \quad (26)$$

$$0 = a_2 E + \frac{\beta S(B+P)}{N} - (\alpha + \gamma + \mu)B; \quad (27)$$

$$0 = \alpha B + \delta V + kD - (\theta + \varepsilon + \mu)P; \quad (28)$$

$$0 = \theta P - (\sigma + \mu)R; \quad (29)$$

$$0 = \gamma B + \sigma R - (\delta + \eta D + \mu)V; \quad (30)$$

$$0 = \rho(B+P) - (k + \phi + \mu)D; \quad (31)$$

From equation (26);

$$E^* = \frac{a_1}{(a_2 + \mu)} S^*; \quad (32)$$

From equation (31);

$$D^* = \frac{\rho}{(k + \phi + \mu)}(B^* + P^*); \quad (33a)$$

From equation (28);

$$0 = \alpha B + \delta V + kD - (\theta + \varepsilon + \mu)P; \quad (33b)$$

Substitute (33) into (28) and rearranging, we have;

$$P^* = \frac{\alpha B^* + \delta V^* + kD^*}{(\theta + \varepsilon + \mu)}; \quad (34)$$

From equation (29);

$$R^* = \frac{\theta P^*}{(\sigma + \mu)}.; \quad (35)$$

From equation (30);

$$V^* = \frac{\gamma B^* + \sigma R^*}{(\delta + \eta D^* + \mu)}; \quad (36)$$

Then, substituting Equations (33) and (35) into (36), we obtained;

$$V^* = \frac{\left[\gamma B^*(\sigma + \mu) + \sigma\theta P^*\right](k + \phi + \mu)}{(\sigma + \mu)\left[\eta\rho(B^* + P^*) + \delta(k + \phi + \mu)\right]}; \quad (37a)$$

From equation (25);

$$S^* = \frac{\Lambda}{(a_1 + \mu)}; \quad (37b)$$

However, letting $c_1 = \alpha + \gamma + \mu,\ c_2 = \theta + \varepsilon + \mu, c_3 = \sigma + \mu, c_4 = \delta + \mu, c_5 = k + \phi + \mu$.
And now substitute Equations (37) and (33) into equation (34) and simplifying further, result in;

$$\alpha B^* + \delta\left(\frac{\gamma B^*}{c_4} + \frac{\sigma\theta P^*}{c_3 c_4}\right) + k\left(\frac{\rho(B^* + P^*)}{c_5}\right) - c_2 P^* = 0; \ldots \quad (38)$$

Collecting the terms for $B^* + P^*$ in (38), we have;

$$B^*\left(\alpha + \frac{\delta\gamma}{c_4} + \frac{k\rho}{c_3 c_4}\right) = P^*\left(c_2 - \frac{\delta\sigma\theta}{c_3 c_4} - \frac{k\rho}{k_5}\right); \ldots \quad (39)$$

This gives us a fixed ratio as; $M = \frac{P^*}{B^*} = \frac{\left(\alpha + \frac{\delta\gamma}{c_4} + \frac{k\rho}{c_3 c_4}\right)}{\left(c_2 - \frac{\delta\sigma\theta}{c_3 c_4} - \frac{k\rho}{k_5}\right)}; \ldots$ (40)

Meanwhile, from equation (27);

$a_2 E + \frac{\beta S(B^* + P^*)}{N} - c_1 B^* = 0$. Since $P^* = MB^*$. Then, we substitute into Equation (27), to achieve Equation 41 as thus;

$$a_2 E + \frac{\beta S B^*(1+M)}{N^*} - c_1 B^* = 0; \ldots \quad (41)$$

Assuming N* is the total population at equilibrium (N* = S* + E* + all gambling classes), we can define a constant W such that (N* = S* + E* +WB*), where W accounts for all the proportions of P, R, V and D relative to B. However, ssubstituting N^* into Equation (41) results in a quadratic Equation for B^*, as; $A(B^*)^2 + B(B^*) + C = 0$;

Where $A = -c_1 W$, $B = a_1 E^* W + \beta S^*(1+M) - c_1(S^* + E^*)$, $A = -a_2 E^*(S^* + E^*)$

It is necessary to know that, the positive root of this quadratic equation gives the value of B^*.
Therefore, the coordinate of the equilibrium E_1 is;

$S^* = \frac{\Lambda}{(a_1 + \mu)}$, $E^* = \frac{a_1}{(a_2 + \mu)} S^*$, B^* = positive root of the quadratic equation, and $P^* = MB^*$,

$R^* = \frac{\theta P^*}{(\sigma + \mu)}$, $V^* = \frac{\left[\gamma B^*(\sigma + \mu) + \sigma\theta P^*\right](k + \phi + \mu)}{(\sigma + \mu)\left[\eta\rho(B^* + P^*) + \delta(k + \phi + \mu)\right]}$, $D^* = \frac{\rho}{(k + \phi + \mu)}(B^* + P^*)$.

Global Stability of the Gambling-Free Equilibrium

Considering the gambling model with respect to the state vector $X(t) = (S, E, B, P, R, V, D)^T$ it is defined in a positively invariant region, $\Omega = \left\{X \in R_+^7 : N(t) \le \frac{\Lambda}{\mu}\right\}$. Recalling that the gambling-free equilibrium (GFE) is given by $G_0 = \left[\frac{\Lambda}{(a_1 + \mu)}, \frac{a_1 \Lambda}{(a_1 + \mu)(a_2 + \mu)}, 0,0,0,0,0\right]$.

Then, we now have defined the Lyapunov function as;

$$L(t) = b_1 E(t) + b_2 B(t) + b_3 P(t); \ldots \quad (42)$$

where $b_1, b_2, b_3 > 0$ are constants to be chosen appropriately.
By differentiating Equation (42) with respect to ‘t’, the we arrived at Equation 43 as;

$$\frac{dL}{dt} = b_1 \frac{dE}{dt} + b_2 \frac{dB}{dt} + b_3 \frac{dP}{dt}; \ldots \quad (43)$$

Furthermore, substituting the model Equations (2), (3) and (4) into Equation (43), gave;

$$\frac{dL}{dt} = a_1 b_1 S - b_1(a_2 + \mu)E + b_2 a_2 E + b_2 \frac{\beta S(B+P)}{N} - b_2(\alpha + \gamma + \mu)B + b_3 \alpha B - b_3(\theta + \varepsilon + \mu)P; \ldots \quad (44)$$

Again, to obtain the Lyapunov coefficients, then let;

$$b_1 = \frac{a_2}{(a_2+\mu)},\ b_2 = 1,\ b_3 = \frac{\alpha}{\theta+\varepsilon+\mu};\ \text{.................................} \quad (45)$$

Also, further substituting Equation (45) into (44) and rearranging terms resulted to;

$$\frac{dL}{dt} \le \left[\frac{\beta S}{N}\left(\frac{1}{(\alpha+\gamma+\mu)}\frac{1}{(\theta+\varepsilon+\mu)}\right)-1\right]\left((\alpha+\gamma+\mu)B+(\theta+\varepsilon+\mu)P\right).$$

However, using the invariant region within $\Omega = \frac{S}{N} \le \frac{S^*}{N^*}$,

Therefore, $\frac{dL}{dt} \le (R_e - 1)((\alpha+\gamma+\mu)B + (\theta+\varepsilon+\mu))$ and;

where $R_e = \frac{\beta S}{N}\left(\frac{1}{(\alpha+\gamma+\mu)}\frac{1}{(\theta+\varepsilon+\mu)}\right)$

Importantly, taking into consideration the sign of the Lyapunov derivative, we have arrived at;

Case 1: If $R_e < 1$; then, $\frac{dL}{dt} \le 0$, for all $X \in \Omega$. Therefore, $\frac{dL}{dt} = 0$; Also,

We let $M = \left\{X \in \Omega : \frac{dL}{dt} = 0\right\}$, as $M = \{B = P = 0\}$.

Then, substituting $B = P = 0$ into the model Equation (1) to (7) gives;

$$\frac{dE}{dt} = -(a_2+\mu)E,\ \frac{dR}{dt} = -(\sigma+\mu)R,\ \frac{dV}{dt} = -(\delta+\mu)V,\ \frac{dD}{dt} = -(\phi+\mu)D.$$

Hence, as $E, R, V, D \to 0$, and $t \to \infty$, this resulted to $S(t) = S^*$. Therefore, the largest invariant set in M is given as the singleton $\{G_0\}$. By applying the LaSalle's invariance principle, it common that every solution with initial condition in Ω approaches G_0 as $t \to \infty$. Conclusively, the global stability result establishes a strong theoretical guarantee that once control strategies reduce the effective reproduction number below unity, gambling behaviour will eventually disappear from the youth population, even in the presence of digital exposure and financial stresses. This therefore elevates the model beyond standard gambling models and confirms its policy relevance for Nigeria's policy makers and the Nigerian governments at all levels.

Global Stability of the Endemic Gambling Equilibrium

Here, we derived *Theorem 3:*

That is, if $R_e > 1$, then the endemic gambling equilibrium of the gambling model is globally asymptotically stable in Ω.

Proof: To do this, we let the gambling model be defined on the positively invariant region;

$\Omega = \left\{(S, E, B, P, R, V, D) \in R_+^7 : N \le \frac{\Lambda}{\mu}\right\}$; thus, assuming that $R_e > 1$, then endemic gambling equilibrium occurs and is given by $E_1 = (S, E, B, P, R, V, D)$, $B^* > 0$, $P^* > 0$.

To establish global asymptotic stability (GAS) of E_1, we construct a Volterra-type Lyapunov function

$L(t) = \sum_{X \in \{S,E,B.P,R,V,D\}} \left(X - X^* - X^* In\frac{X}{X^*}\right)$ with $L \ge 0$ for all $X > 0$, $L = 0$ if and only if $X = X^*$ and finally 'L' is radially unbounded in Ω.

Taking the time derivative of the Lyapunov function, we obtained;

$$\frac{dL}{dt} = \sum_X \left(1 - \frac{X^*}{X}\right)\frac{dX}{dt};\ \text{...} \quad (46)$$

At equilibrium E_1, substitute equation (25) to (31) into (46), and cancel all the linear terms, the resulting simplification will be;

$$\frac{dL}{dt} = -\sum_X \mu X^*\left(\frac{X^*}{X} - 1\right)^2 - \frac{\beta S^*(B^* + P^*)}{N^*}\left(\frac{S}{S^*} - \frac{B+P}{B^*+P^*}\right)^2 \le 0;\ \text{.............................} \quad (47)$$

Thus, $\frac{dL}{dt} \le 0$ for all $X \in \Omega$.

Let $M = \left\{ X \in \Omega : \frac{dL}{dt} = 0 \right\}$

From equation (47), $\frac{dL}{dt} = 0 \Leftrightarrow \frac{X}{X^*} = 1$, for all compartments. Hence, $M = \{E_1\}$.

Hence, applying LaSalle's invariance principle, since Ω is positively invariant and compact, L is non-increasing, and the largest invariant set in as; $\left\{ \frac{dL}{dt} = 0 \right\}$; is E_1. Therefore;

$$\lim_{t \to \infty} X(t) = E_1 \text{ for all } X \in \Omega.$$

Convincingly, the global stability of the endemic gambling equilibrium confirms that gambling persistence is inevitable once the effective reproduction number exceeds unity. The result shows that the threshold driven nature of gambling dynamics in Nigeria and mathematically requires justifiable aggressive regulatory and recovery-focused interventions.

3. Results

Sensitivity Analysis of the Effective Reproduction Number

Sensitivity analysis is used to determine which parameters most strongly influence the effective reproduction number Re, which governs whether gambling behaviour persists or dies out in the population. The normalized forward sensitivity index is adopted because it provides a dimensionless and policy-relevant measure of impact [7, 8]. Let 'u' be a function of parameter 'p'. Then, the normalized forward sensitivity index of 'u' with respect to 'p' is

$$\gamma_u^p = \frac{\partial u}{\partial p} \cdot \frac{p}{u}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(48)$$

This index measures the percentage change in 'u' resulting from a 1% change in 'p' [8]. Carrying out the sensitivity analysis on (23) using the parameters values in Table 3 gives the sensitive indices of the respective parameters in Table 4.

Table 3: Parameter Values of the Effective Reproduction Number

Parameters	Value	Source
β	0.4yr^{-1}	[9]
α	0.2 yr^{-1}	[10]
γ	0.15 yr^{-1}	Assumed
θ	0.25 yr^{-1}	[11, 12]
ε	0.10 yr^{-1}	[2]
μ	0.02 yr^{-1}	[13]
S^*/N^*	0.62	[13]

Table 4: Result of Sensitivity Index on Parameter

Parameters	Sensitivity indices
β	+1.00
α	-0.29
γ	-0.22
θ	-0.31
ε	-0.12
μ	-0.054

From Table 4, the normalized forward sensitivity analysis reveals that peer influence (β) and rehabilitation effectiveness (θ) are the most critical determinants of gambling persistence in Nigeria. Evidence-based interventions targeting these parameters can decrease the effective reproduction number below unity, leading to the long-term eradication of endemic gambling behaviour. The numerical sensitivity graph plots showing the effect of each parameter on the effective reproduction number are shown in Figures 2 to 8.

Numerical Simulation of the Endemic Gambling Equilibrium

The numerical experiment investigated how key sensitive parameters identified through normalized forward sensitivity analysis affect the endemic gambling equilibrium of the model Equation (1) to (7). The results have focused on the peer/media influence rate (β) and the rehabilitation rate (θ), since these parameters exhibited the largest sensitivity indices with respect to the effective reproduction number Re. The system was evaluated numerically utilizing a fourth-order Runge-Kutta methodology over a long-time interval to ensure convergence to equilibrium. Initial conditions used are (S(0), E(0), B(0), P(0), R(0), V(0), D(0)) = (500, 80, 50, 30, 20, 40, 10) with parameter value of β and θ from table 3 and keeping all other parameters constant. Figures 9 and 10 show the resulting graphical plots of the effect of each parameter on the endemic gambling equilibrium.

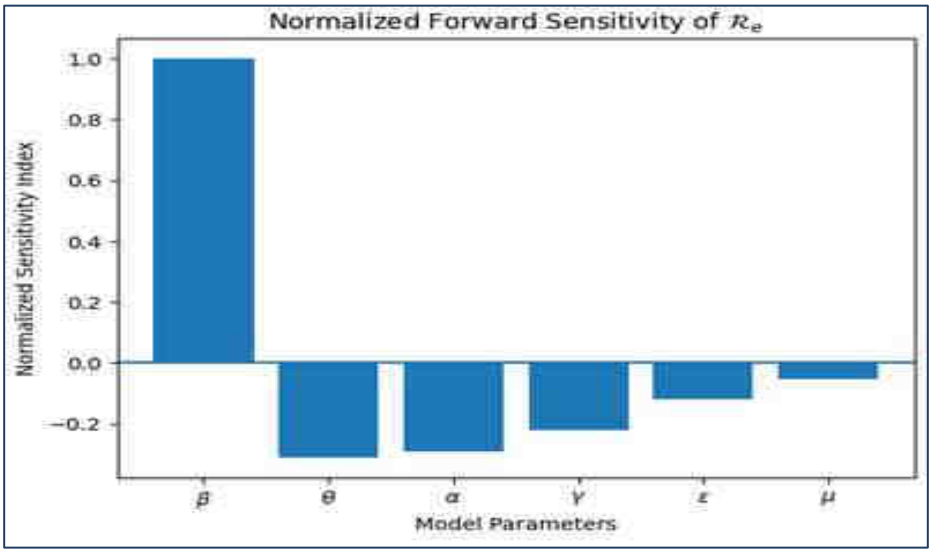

Figure 2: Bar Chart of the Normalized Forward Sensitivity of R_e on the Model Parameters.

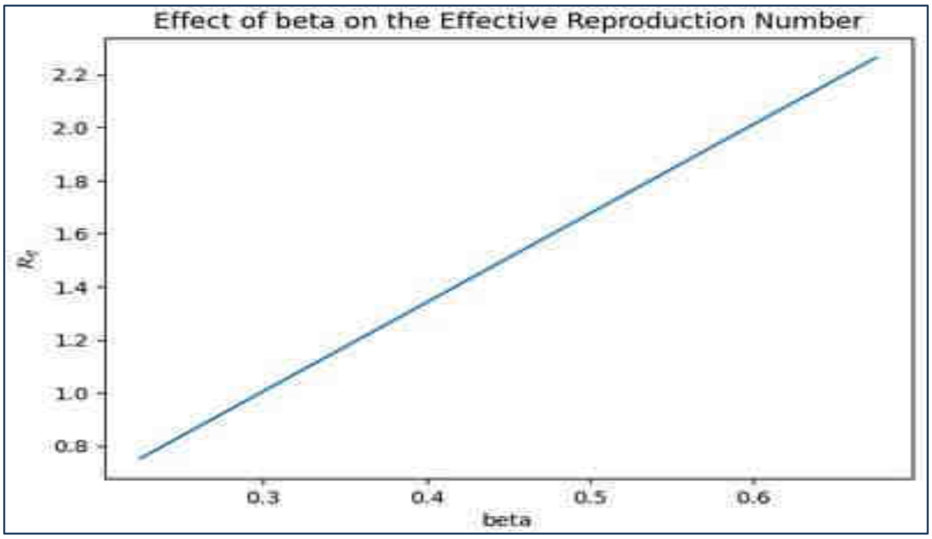

Figure 3: Effect of β on the Effective Reproduction Number

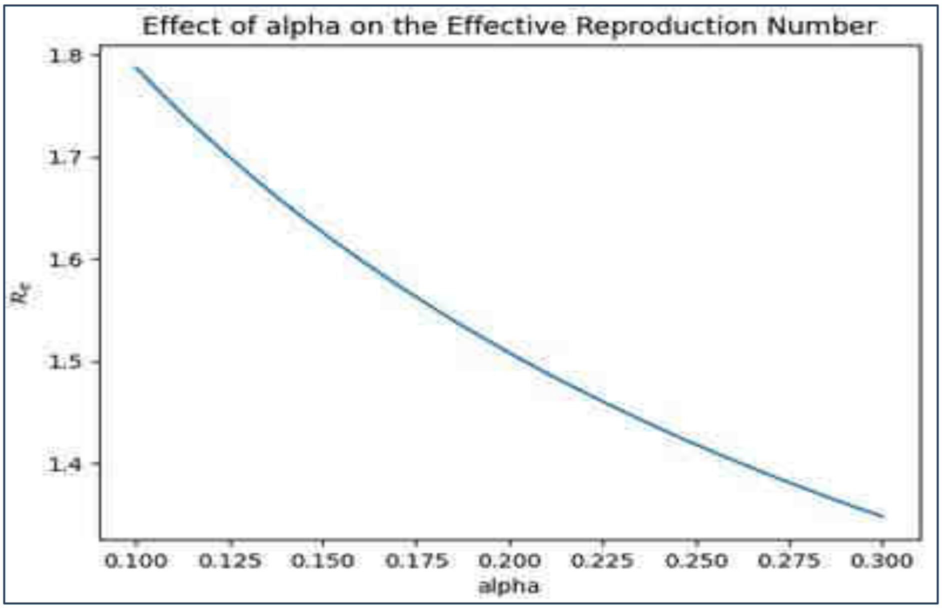

Figure 4: Graph of Effect of Alpha (α) on the Effective Reproduction Number

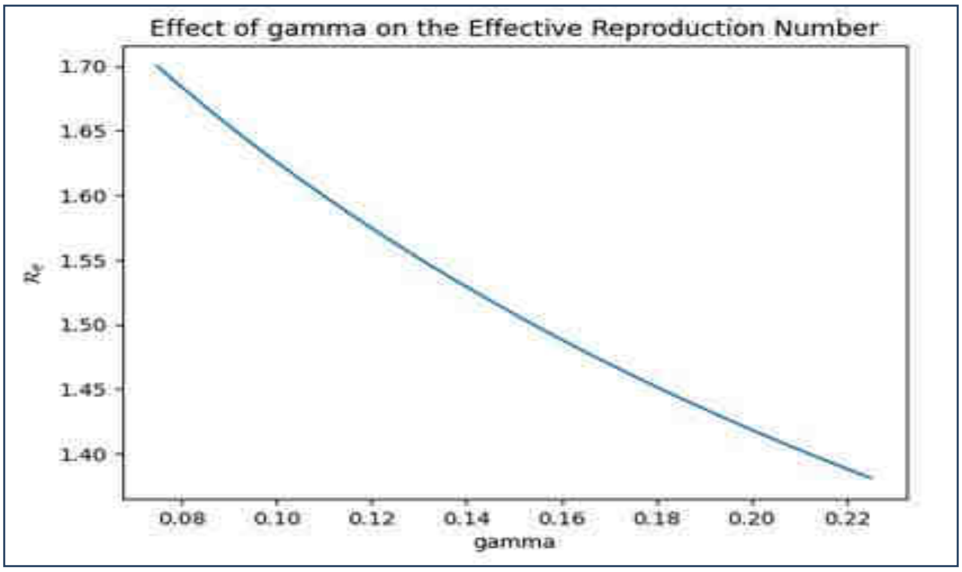

Figure 5: Effect of Gamma (γ) on the Effective Reproduction Number

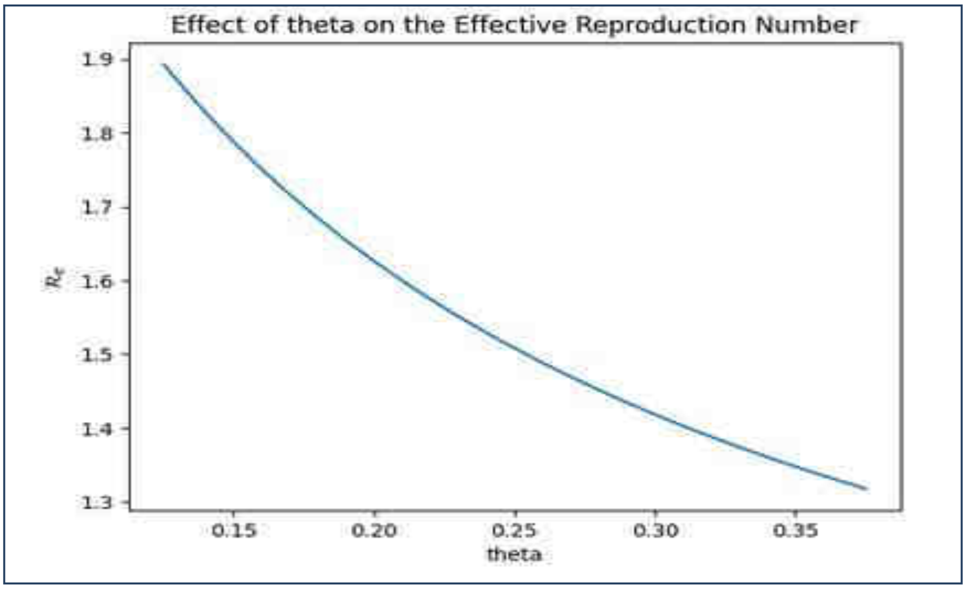

Figure 6: Effect of θ on the Effective Reproduction Number [14, 15]

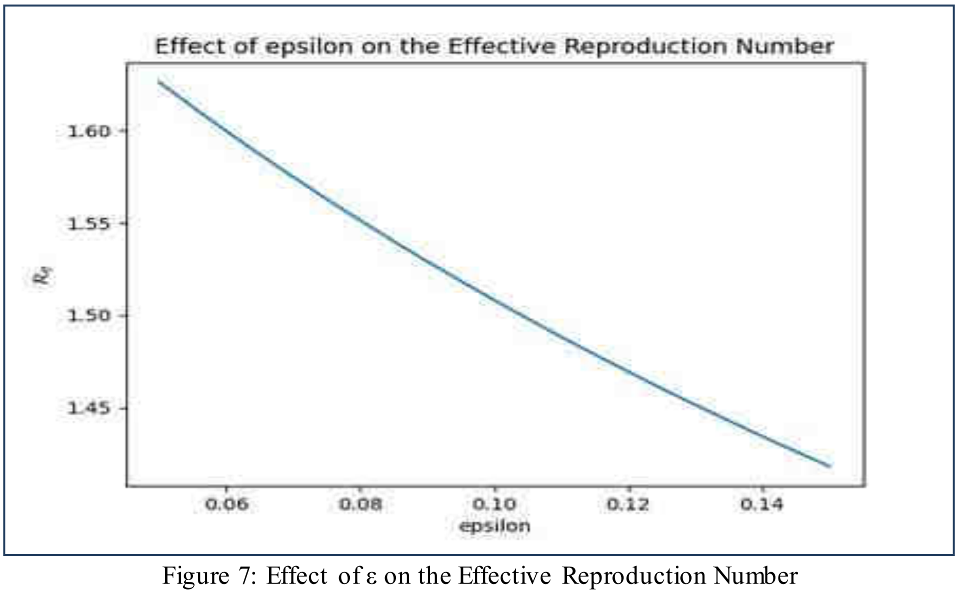

Figure 7: Effect of ε on the Effective Reproduction Number

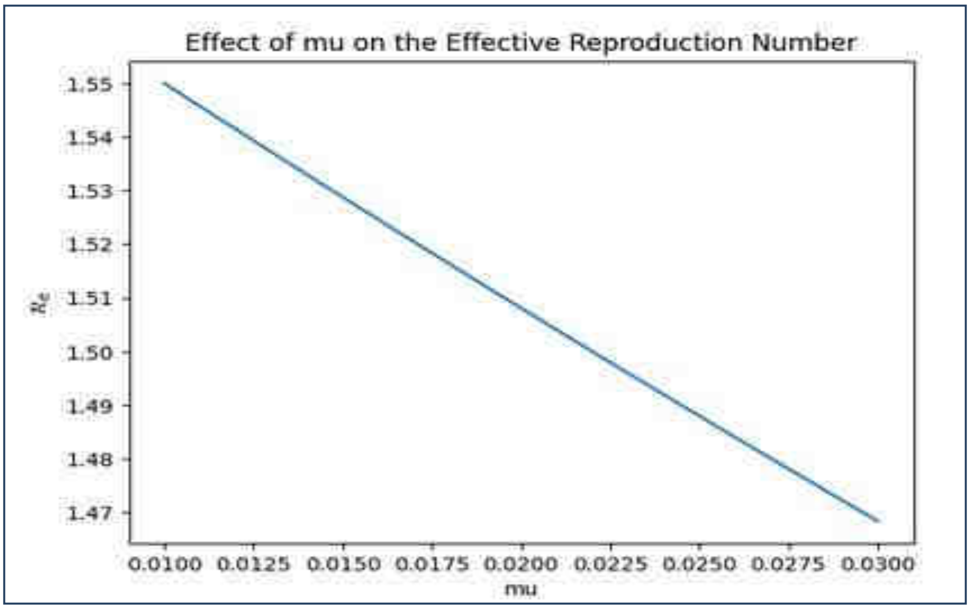

Figure 8: Effect of μ on the Effective Reproduction Number

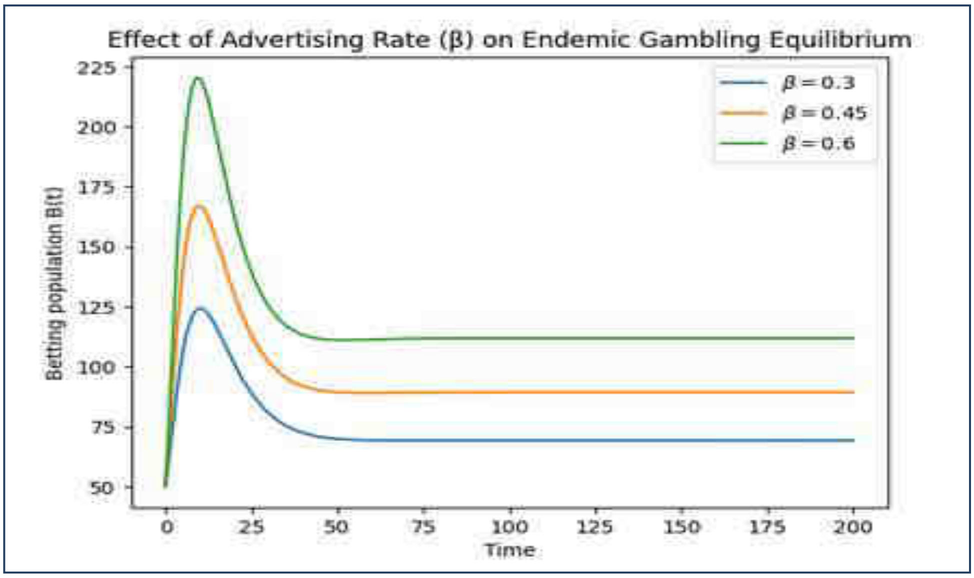

Figure 9: Effect of β on the Endemic Gambling Equilibrium

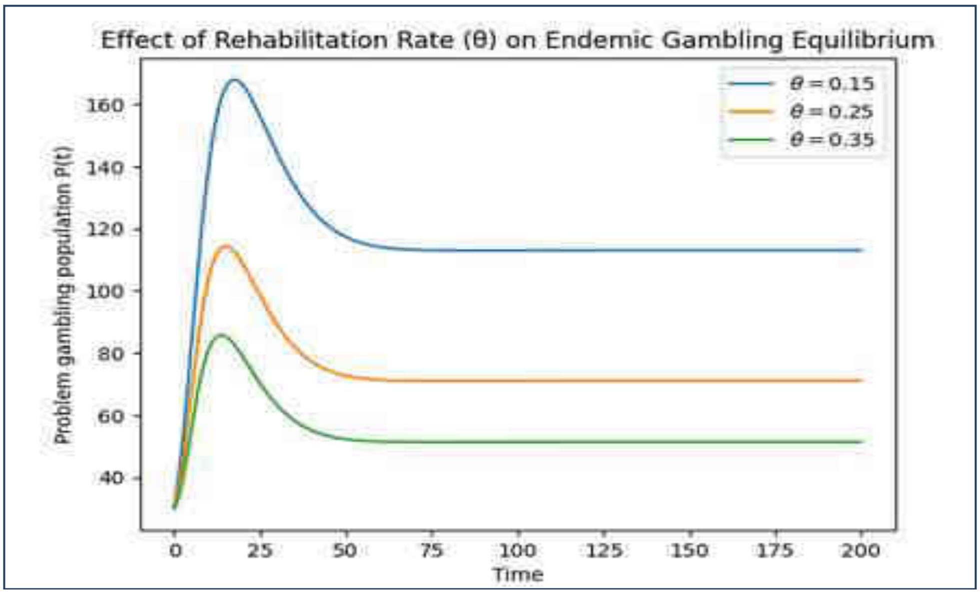

Figure 10: Effect of θ on the Endemic Gambling Equilibrium [14,15,16]

4. Discussion

Figure 2: Bar Chart of the Normalized Forward Sensitivity of R_e on the Model Parameters

This Figure is made up of positive bars (β) and Negative bars (θ, α, γ, ε, μ). The positive bar (β) has a sensitivity index of +1, which shows that peer/media influence has a direct and proportional effect on gambling spread. Any decrease in betting advertisements results in an equal percentage reduction in revenue. While the negative bars (θ, α, γ, ε, μ) parameters reduce the effective reproduction (Re) when increased.

Figure 3: Effect of Peer/Media influence Rate (β)

Figure 3 shows that Re increases linearly with β. This confirms the analytical result of the sensitivity index Re = 1. Any proportional reduction in betting advertisements or peer influence yields an equal proportional reduction in gambling spread. Advertising bans and restrictions on betting sponsorships are the most effective control measures.

Figure 4: Effect of Addiction Progression Rate (α)

Figure 4 shows that Re decreases monotonically as α increases. Delay progression into addiction reduces the time individuals spend generating new gamblers. Educational interventions and delaying addiction start are effective.

Figure 5: Effect of Quitting Rate (γ).

Figure 5 shows that increasing γ produces an obvious decline in R_e. Self-exclusion programs and social support can significantly reduce gambling persistence.

Figure 6: Effect of Rehabilitation Rate (θ).

Figure 6 shows that θ has one of the **steepest negative slopes.** Improving access to rehabilitation produces a large reduction in gambling transmission. This aligns with the high sensitivity magnitude obtained in Table 4.

Figure 7: Effect of Relapse Resistance (ε).

Figure 7 shows that increasing ε reduces Re, though at a slower rate. Relapse prevention remains important but is secondary to rehabilitation and advertising control.

Figure 8: Effect of Natural Exit Rate (μ)

Figure 8 shows that Re declines only slightly with increasing μ. The demographic turnover alone is insufficient to control gambling. It has a very small sensitivity value of Re = -0.054.

Figure 9: Effect of Advertising/Peer Influence rate (β)

Figure 9 shows that increasing β significantly increases the long-term equilibrium level of the betting population B(t). For all values of β, the solutions converge to a positive endemic equilibrium, confirming analytical results. This means that higher exposure to betting advertisements and peer influence leads to faster initial growth of betting behaviour and a higher endemic gambling burden. Hence, strict regulation of gambling advertisements and sponsorships is essential for reducing endemic gambling levels.

Figure 10: Effect of Rehabilitation Rate (θ)

Figure 10 shows that increasing θ significantly reduces the equilibrium level of problem gamblers P(t). Higher rehabilitation rates also reduce transient peaks, limiting the period of severe gambling harm. Therefore, rehabilitation reduces the average duration of problem gambling, and the system still converges to an endemic equilibrium, but at a very low level, illustrating strong control potential.

5. Conclusion and Recommendation

5.1 Conclusion

This investigation developed and examined the comprehensive compartmental mathematic model to study the dynamics of youth gambling in Nigeria, explicitly incorporating digital exposure, financial debt accumulation, relapse vulnerability, and policy enforcement mechanisms. Unlike earlier gambling models that focus primarily on betting participation and addiction, the proposed extended framework captures the full socio-economic feedback loop sustaining gambling behaviour among Nigerian youth. The model design splits the population into various classes including the susceptible, exposed, recreational bettors, problem gamblers, recovering individuals, vulnerable individuals, and debt-burdened individuals. This structure allowed the explicit modelling of digital advertising and peer influence, financial distress, and post-recovery relapse, which the findings of this work have shown that they are particularly relevant in Nigeria's rapidly expanding online betting environment. Moreso, the mathematical properties of the model, including positivity of solutions and invariant region, the gambling-free equilibrium have been shown to exist, and the explicit expression for the effective reproduction number Re has been derived using the next-generation matrix approach. This threshold quantity has been identified as the variable that governs the initial spread of gambling behaviour through peer and digital exposure.

Furthermore, the local and global stability analysis revealed that the gambling-free equilibrium is stable when Re < 1. The existence and stability of an endemic gambling equilibrium were analytically established and further validated through numerical simulations. These simulations showed that betting B(t) and problem gambling P(t) converge to positive equilibrium levels under realistic parameter values, confirming the long-term persistence of gambling among youth when structural drivers remain unaddressed. The numerical results also illustrated how increased digital exposure (β) increases gambling prevalence, while rehabilitation (θ) and recovery programs reduce but do not eliminate gambling in the presence of relapse and debt pressure. Additionally, the normalized forward sensitivity analysis of Re identified the peer and digital influence rate (β) as the most influential parameter driving gambling spread, followed by rehabilitation (θ) and addiction progression rates (α). In contrast, the natural demographic exit (μ) has been shown to have the least impact. These findings therefore, quantitatively have revealed that youth gambling in Nigeria is policy-driven rather than demographically driven, emphasizing the importance of regulatory and behavioural interventions as prominent measures to drastically reduce the spreading and sustenance of such activities among the youths.

5.2 Recommendation

Based on the results of the model, it is recommended that policymakers in Nigeria adopt a multi-pronged intervention strategy to effectively control youth gambling. First, strict regulation and monitoring of digital gambling advertisements and online betting platforms should be prioritized, as peer and digital exposure rate were identified as the most influential drivers of gambling spread. Secondly, expanded access to youth-centred rehabilitation and counselling services is essential to reduce the progression into problem gambling and lower long-term gambling prevalence. Thirdly, financial debt mitigation measures, including betting limits, debt counselling, and restrictions on credit-based betting, should be integrated into gambling control policies to break the relapse debt feedback loop identified in the model. Finally, strong policy enforcement and compliance mechanisms must accompany these interventions, and an integrated regulatory, financial, and public health approach is therefore necessary to achieve sustainable reductions in youth gambling in Nigeria.

5.3 Contribution to Knowledge

The present study is believed to contribute to knowledge as it is providing a firsthand comprehensive mathematical framework that integrates digital exposure, financial debt, and policy enforcement into youth gambling dynamics in Nigeria. The results which have emphasized significantly that effective control of youth gambling requires multi-layered interventions rather than isolated policies, is additional exceptional knowledge that will be of great assistance to the policy makers and to the various levels of governments in the development of policies and strategies for their implementations to curb gambling activities among the productive youths in Nigeria.

5.4 Area of Consideration for Future work

Future work may extend this framework to include age-structured dynamics, spatial heterogeneity, stochastic effects, and optimal control strategies to further inform evidence-based gambling regulation and public health policy in Nigeria.

Authors' Contributions

Mutah Wadai conceptualized the ideas, composed and developed the topic, and also supervised the assembling of the paper's manuscript. On his part, Ibekwe John Jacob executed the tasks of designing the study model, the simulation and data analysis, and the interpretation of the properties of the utilized model while Idongesit Nnammonso Akpan executed the tasks of preparing and compiling the manuscript in a journal format, type setting, editing, and the provision of correct and adequate references, as well as the production of the final drafted form of the paper's manuscript. Then, Adem Kilicman performed the plagiarism checking, critical assessment of the drafted paper's manuscript and also offered useful suggestions to further enhanced the quality of this work.

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Data Availability

All the data utilized to support the findings of this study will be made readily accessible by the corresponding author upon a reasonable request.

Consent and Ethical Approval

In this work, no consent is sorted but all the sources of secondary data found in the public domain that are used in this work have been acknowledged and therefore this paper has no ethical violation.

Declarations of Conflict of Interests

All authors involved in this study declare that they have no known existed competing interest.

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