RESEARCH ARTICLE

JENER Journal of Empirical and Non-Empirical Research
Volume 2, Issue 3, Page 549-551 | Article No. 76
DOI: 10.4898/jener.v2i3.a76
ISSN: 7895-9216
Journal homepage: https://jenerjournal.com
Journal email: editor@jenerjournal.com

JENER Journal
JENER Journal of Empirical and
Non-Empirical Research

Displaying the Numerical Sequence of a 16-Dimensional Binary System

Mustafa H. Musa[1] & Sajjad Muhammad M.[2]

[1]Department of Physics, College of Education, Mustansiriyah University, Baghdad, Iraq
[2]Department of Medical Physics, College of Science , Hilla University College, Babylon, Iraq

ARTICLE INFORMATION
Article history:
Published: March 2026

Keywords:
Mathematical abstraction
X-gate
Qubit state , Hilbert's space
,Manipulation state

ABSTRACT
A brief study of qubit state manipulation in a {0, 1} quantum system consisting of four entangled qubits using an X-gate. Following a randomized order of qubit targeting reveals randomness in the dimensions of the four-qubit state array. The optimal gate targeting sequence was analyzed by considering each manipulation state as a set of qubit states, and a suitable metric for understanding the study's results was proposed based on changing the position of the column array elements.

1. Introduction

In quantum circuits, the qubit states are the input value of the circuit, and since the qubit states are described by Hilbert space, the inflation of that space is achieved through ⊗ (Tensor produc). The state of a qubit in a circuit will only have value through a Hadmard gate, leading to superposition, meaning the circuit's output consists of two states ($|state\rangle + |state\rangle$), and the presence of a CNOT gate determines whether these states are entangled or not[1,],[2]. The state of a qubit is simply a column matrix $\psi \in \mathcal{H}^{2\times1}$ When the circuit consists of 4- qubits, the state of the qubit $\psi \in \mathcal{H}^{2\times16}$,this is because of ⊗ , The aim of the study will depend on the 16 - dimensions of the array and when targeting these four qubits by the X-gate is the least random [3],[4].

2. Methodology

The mathematical relationship of qubit state :

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix}\alpha\\0\end{bmatrix} + \begin{bmatrix}0\\\beta\end{bmatrix} \tag{1}$$

The x-gate : A square matrix that works on a single qubit ,

$$X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

It is an operator that can be used on a qubit state :

$$X|\psi\rangle = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}\alpha\\0\end{bmatrix} + \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0\\\beta\end{bmatrix} = \beta|0\rangle + \alpha|1\rangle$$

α, β These are the amplitudes of the qubit states, which represent the probability of each state [5],[6],[7]. If the arrangement of quantum circuits is (q_0 ,q_1, q_2, q_3) , The Hadramid Gate targets the third qubit (q_2) and creates a quantum superposition state[8]. Therefore, targeting this qubit with the X-gate will disappear. Since the circuit consists of four qubits, we will have 16 possible states on the basis vector of the states, and we have a superposition state $(q_0\ q_1 q_2 q_3)$ +$(q_0\ q_1 q_2 q_3)$, so the total output of the circuit is 8-states. Given that the states in a state of quantum entanglement are inseparable states, which theoretically apply to the binary system {0,1}[9],[10]. The emergence of entanglement states will be adopted as follows [11] :

$$\psi_1 = (0,0,0,0) + (1,1,1,1)\ , \qquad \alpha, \beta = {}^{1}\!/_{\sqrt{2}}, |\alpha|^2 + |\beta|^2 = 1 \ \text{(normalization)}$$

$$\phi = \{\quad\} \Rightarrow \nexists X - gate\ \ , \forall\ qubit \in \chi$$

$$\chi = \{\{\,\}, \{\psi_1\}, \{\psi_1, \psi_2\}, \{\psi_1, \psi_2, \psi_3\}, \{\psi_1, \psi_2, \psi_3, \psi_4\}, \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5\},$$
$$\{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6\}, \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7\}, \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8\}\}$$

The input state of the circuit : $\Lambda_{10} \subseteq \psi_1 = \Lambda_{io} + \Lambda_{i1}$
So,

$$\mathcal{O}ch = \{\{\,\}, \{\psi_1\}, \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5\}, \{\psi_1, \psi_2, \psi_3\}, \{\psi_1, \psi_2\}, \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6\},$$
$$\{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7\}, \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8\}, \{\psi_1, \psi_2, \psi_3, \psi_4\}\}$$

Let : $\psi_1 \in \mathbb{R}^{16}, \Lambda_{io} \subseteq \psi_1$

$$\Lambda_{io} \neq \phi \Rightarrow \Lambda_{io} \bigcap \psi_1 = \Lambda_{i1} \subseteq \psi_1 \in \mathcal{O}ch$$

$$\Rightarrow \Lambda_{io} \bigcap \chi = {}^{Och}/_{\Lambda_{io}} \subseteq Och \; \forall \psi_1$$

$$Och \neq \chi \Rightarrow \forall \, Och \bigcup \chi = \{\psi_i | i = 1, \dots, 8\}; \; \frac{\chi, Och}{\{\{\,\}, \{\psi_1\}\}}$$

- $\{\,\}, \{\psi_1\} \in \chi \wedge \{\,\}, \{\psi_1\} \in Och$
- $\{\psi_1, \psi_2\} \in \chi \wedge \{\psi_1, \psi_2\} \subseteq \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5\} \in Och$
- $\{\psi_1, \psi_2, \psi_3\} \in \chi \wedge \{\psi_1, \psi_2, \psi_3\} \in Och$
- $\{\psi_1, \psi_2, \psi_3, \psi_4\} \in \chi \wedge \{\psi_1, \psi_2, \psi_3, \psi_4\} \supseteq \{\psi_1, \psi_2\} \in Och$
- $\{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5\} \in \chi \wedge \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5\} \subseteq \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6\} \in Och$
- $\{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6\} \in \chi \wedge \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6\} \subseteq \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7\} \in Och$
- $\{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7\} \in \chi \wedge \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7\} \subseteq \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8\} \in Och$
- $\{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8\} \in \chi \wedge \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8\} \supseteq \{\psi_1, \psi_2, \psi_3, \psi_4\} \in Och$

The manipulation did not change ψ_1, but it made $\{\psi_1, \psi_2\} \leftrightarrow \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5\}$. We will observe the value of the scale in the next section, which will make the mathematical abstraction into numerical values that we can observe.

3. Discussion

To add a necessary dimension to manipulating the outputs of the quantum circuit, we propose following the function ($e^{a_i/n-1}$) to show the growth of the studied change plus the information limit $[ln]$, Where $a_i\{i = 1,2, \dots . n\}$ is a position number of the qubit state vector, which represents the state of 4- qubits (n=2^4).To determine the best manipulation using the x-gate, we will compare the numerical value of the input vector ($\lambda(\psi_0 = |0000\rangle), a_i = 1$) with the numerical value of the initial entangled output state of a four-qubit circuit using the following equation , which will show which qubit target is appropriate without distortion and drift of the computational state vector :

$$\lambda(\psi_i) = e^{\frac{a_i}{n-1}} + ln\left[\frac{n(n-1) - a_i}{2(a_i + (n-1))}\right] \tag{3}$$

Table 1. represents the results of the deliberate manipulation of the X-gate (och) in the circuit, where the input value is $\lambda(\psi_0)$ =3.20162. We observe from FIG 3. that the X-gate line targets the qubits in FIG 1. using the method of untargeted qubit sequence {without gate, $q_3, q_1, q_0, (q_3, q_1), (q_3, q_0), (q_1, q_0), (q_3, q_1, q_0)$}.

TABLE 1. Numerical results for optimal manipulation using the X-gate

x	ψ_i	$\lambda(\psi_i)$	$\Delta = \lvert\lambda(\psi_i) - \lambda(\psi_0)\rvert$	x-gate
1	ψ_1	7.4608	4.25918	without
2	ψ_5	7.32012	4.1185	q_0, q_1
3	ψ_3	7.20107	3.99945	q_1
4	ψ_2	7.1029	3.90128	q_0
5	ψ_6	7.02505	3.82343	q_0, q_3
6	ψ_7	6.96705	3.76543	q_1, q_3
7	ψ_8	6.92857	3.72695	q_0, q_1, q_3
8	ψ_4	6.90939	3.70777	q_3

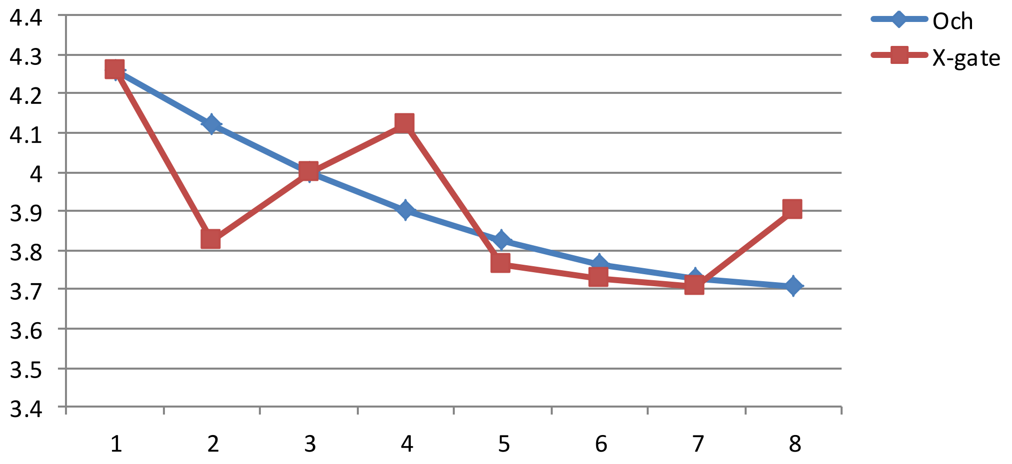

FIG 1. Och(optimal change for state) gives a transition of 1 to the binary vector {0,1} using the X-gate.

4. Conclusion and Recommendations

What we obtained in $\mathcal{O}c\hbar$ must be in the same system we dealt with because the states of qubits depend on how they are entangled and superimposed. Mathematical abstraction was considered an analysis of what is most suitable for manipulation, as shown in FIG 1. Manipulating the elements of a matrix in a physical system may be random in order to achieve the most suitable outcome.

References

[1] Yanofsky, N. S., & Mannucci, M. A. (2008). *Quantum computing for computer scientists*. Cambridge: Cambridge University Press. pp. 138–144. https://doi.org/10.1017/CBO9780511813887.

[2] National Academies of Sciences, Engineering, and Medicine. (2019). *Quantum computing: Progress and prospects*. Washington, DC: National Academies Press. p. 82. https://doi.org/10.17226/25196.

[3] Bataille, M. (2022). *Mathematical structures in computer science*, 32, 257. https://doi.org/10.1017/S0960129522000305.

[4] Scherer, W. (2019). *Mathematics of quantum computing: An introduction*. Switzerland: Springer Nature. Vol. 11, p. 169. https://doi.org/10.1007/978-3-030-12358-1.

[5] Choi, M. S. (2022). *A quantum computation workbook*. Switzerland: Springer Nature Switzerland AG. p. 52. https://doi.org/10.1007/978-3-030-91214-7.

[6] Hughes, C., Isaacson, J., Perry, A., Sun, R. F., & Turner, J. (2021). *Quantum computing for the quantum curious*. Switzerland: Springer. p. 25. https://doi.org/10.1007/978-3-030-61601-4

[7] Kaye, P., Laflamme, R., & Mosca, M. (2007). *An introduction to quantum computing*. USA: Oxford University Press. p. 100. https://dl.acm.org/doi/10.5555/1206629.

[8] Schuld, M., & Petruccione, F. (2018). *Supervised learning with quantum computers*. Switzerland: Springer. Vol. 17, p. 97. https://doi.org/10.1007/978-3-319-96424-9

[9] Portugal, R. (2013). *Quantum walks and search algorithms*. Switzerland: Springer Science & Business Media. Vol. 19, pp. 257–258. https://doi.org/10.1007/978-3-319-97813-0

[10] Scherer, W. (2019). *Mathematics of quantum computing: An introduction*. Switzerland: Springer Nature. Vol. 11, p. 169. https://doi.org/10.1007/978-3-030-12358-1

[11] Sutor, R. S. (2019). *Dancing with qubits: How quantum computing works and how it can change the world*. UK: Packt Publishing Ltd. pp. 234, 283.