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A Cost-Effective Model for Minimization Average Total Inspection (ATI) In Sampling Inspection Plans

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ABSTRACT

In every supply chain where the manufacturer/ producer or retailer/consumer is involved, the use of acceptance sampling plan plays an important role in the inspection of raw materials; semi-finished products and finished products where a decision is taken to either accept or reject inspected lots based on the sample results. Rejected lots are either returned to the producer or subjected to rectifying inspection where all the identified defective units are removed and replaced with non-defective units. However, the Average Total Inspection (ATI), personnel requirement and cost is enormous. This study addresses this problem by developing an economic model that provides optimal sampling plans for Single Sampling plan (SSP) and Double Sampling Plan (DSP). Search algorithm is used to find optimal sampling plans that minimize the Average Total Inspection (ATI) and also provide protection for both the producer and the consumer against producer's risks and consumer's risk. The results indicate that the minimum Average Total Inspection (ATI) values for the Single Sampling Plan (SSP) and Doble Sampling Plan (DSP) were 267.19 and 237.32 units respectively. The corresponding optimal sampling plans were $(n = 201, c = 9)$ for SSP and $n_1 = 96, n_2 = 192, c_1 = 3, C_2 = 11)$ for the DSP. In both cases, the optimal sampling plans satisfied the specified producer's and consumer's risks. These findings underscore the importance of optimal sampling plan in every Acceptance Sampling Inspection in order to provide cost effective inspection solutions for producers and consumers as well as other industry stakeholders involved in a supply chain.

1. Introduction

Manufacturers are continuously seeking effective methods for ensuring that products conform to specified standards while maintaining efficiency in inspection and decision-making processes. One of the most widely used statistical quality control techniques for this purpose is acceptance sampling inspection. Acceptance sampling plans (ASPs) are commonly used in quality control to assess and verify product quality by making acceptance or rejection decisions based on sample drawn from submitted lots. They provide a cost-effective and time-efficient method for both the buyers and suppliers to meet their quality and risk requirements (Wu and Wang, 2024; Wu et al., 2024). According to Aziz, Hasim & Zain, 2021 Acceptance sampling determine the acceptability of a lot without necessitating a complete inspection.

In acceptance sampling theory, both producer's risk and consumer's risk are fundamental considerations in the design of inspection plans. Producer's risk refers to the probability of rejecting a good-quality lot, while consumer's risk represents the probability of accepting a poor-quality lot. An optimal sampling inspection plan should therefore achieve a balance between minimizing inspection effort and satisfying these risk requirements. If the risks are not adequately controlled, the inspection process may either penalize producers unnecessarily or expose consumers to defective products. Hence, the development of optimal sampling plans that satisfy specified risk conditions while minimizing average total inspection has become an important area of research in statistical quality control.

2. Literature Review

Many researchers have made several contributions in the development of optimization and cost model for acceptance sampling inspection, most of the notable ones are as follows: Muhammad and Chang, (2016) proposed goal programming model to determine the optimal number of inspectors based on skills and also minimize cost of inspection in single sampling plan. Fallahnezhad and Ahmadi, (2016) developed optimization model to reduce total losses of the producer and the consumer in acceptance sampling inspection. Mohammad and Ahmad, (2016) proposed an optimization model that reduced the producer and consumer losses where cumulative sum of run length of conforming unit was considered. Fallahnezhad and Qazvini (2016) presented an economic cost model using Maxima Nomination Sampling (MNS) technique to reduce total cost. Mansooreh et al., (2018) developed an economic cost model for the design of Rectifying Single Sampling plan to minimize the expected total losses using Maxima Nomination Sampling (MNS) method in the presence of inspection errors. In the MNS method lots are arranged in different sets and then ranked based on the observed quality characteristics of in order of defect level. Results of the MNS method was compared with the Simple Random Sample (SRS) method, it was found that the average total loss using the MNS method was lower. Kumar (2018) developed an economic cost model for achieving optimal single sampling plan that minimizes total cost and satisfies the producer and consumer quality risk. Yun-Cheng, Cheng and Yi-An (2019) compared the economical design of quality with traditional single sampling plans under total quality cost considerations

Yun-Cheng, Cheng and Yi-An (2019) compared the economical design of quality with traditional single sampling plans under total quality cost considerations Iorkegh and Osanaiye (2022) adjusted

the model developed by Kumar to obtain optimal sampling for Rectifying Single Sampling (RSS) plan with the introduction of inspection error. Marques et al. (2024) introduced an economic design of a single sampling plan for electronic components' reliability testing under exponential distribution, providing cost-effective reliability testing strategies and ensuring electronic devices' quality and performance in various applications

In this paper, an optimization model to minimize Average Total Inspection (ATI) and provide protection for both the producer and the consumer is proposed.

3. Methodology

3.1 Defective units in a sample

If x represents the true number of defective units in a sample n and with a binomial random variable 0 to n. Probability that x defective unit will be observed in a sample n is given as:

$$\mathrm{P}(X=x)=\binom{n}{x}p^x(1-p)^{n-x} \qquad x=0,1,\dots,n \tag{1}$$

where p is the proportion of defective units.

3.2 Single Sampling Plan (SSP)

In a single sampling inspection, a sample is randomly taken from a lot N and based on the result a decision is taken to either accept or reject the lot.

Probability of acceptance(P_a) for single sampling plan is calculated as:

$$P_a=p(x\le c)=\sum_{x=0}^{C}\binom{n}{x}p^x(1-p)^{n-x} \tag{2}$$

where p is the true fraction defective units, N is the lot size while x is the number of defective units in a sample,c is the acceptance number and n is the sample size.

Given that the lot is accepted based on the few defective units in the sample, all observed defective units in the sample are replaced with units that are not defective. The remaining unit that is not inspected is $(N-n)$. The probability that the lot will be accepted is $P_a p(N-n)$ (Iorkegh and Osanaiye 2022). However, if the lot is rejected, all defective units in the sample and in the rejected lot are replaced with non-defective units and the lot will contain no defective units.

Thus, Average Outgoing Quality (AOQ) is given as:

$$AOQ_1=\frac{(N-n)pP_a}{N} \tag{3}$$

Average total inspection (ATI) represents the average number of units inspected in the sample and in the rejected lot and is given as:

$$\mathrm{ATI}=\mathrm{n}+(1-P_a)(N-n) \tag{4}$$

3.3 Double Sampling Plan (DSP)

In Double sampling inspection, a sample of size n_1 is drawn from the lot and inspected, if the defective units in the first sample x_1 is less than or equal to the first acceptance number c_1 the lot is accepted on the first sample. If the defective units in the first sample x_1 is greater than the second acceptance number $c_2$the lot is rejected on the first sample. However, if x_1 greater than c_1 but less than or equal to c_2 ,a second sample of size n_2 is taken. If the combined defectives in the two samples $(x_1+x_2)$is less or equal to c_2 the lot is accepted or else the lot is rejected.

The probability of acceptance on the first sample $\left(P_{a_1}\right)$ is:

$$P_{a_1}=P_r\{x_{1\le}\, c_1\}=\sum_{x_1=0}^{c_1}\binom{n_1}{x_1}p^{x_1}(1-p)^{n_1-x_1}$$

The probability of acceptance on the second sample is:

$$P_{a_2}=\sum_{x_1=c_1+1}^{c_2}\left\{\left[\binom{n_1}{x_1}p^{x_1}(1-p)^{n_1-x_1}\right]\times\left[\sum_{x_2=0}^{c_2-x_1}\binom{n_2}{x_2}p^{x_2}(1-p)^{n_2-x_2}\right]\right\} \tag{4}$$

The probability of final acceptance of the lot is denoted by (P_a) and it is the sum of P_{a_1} and P_{a_2} .The acceptance probabilities depend on the fraction defective units p in the lot. This the probability of final acceptance of the lot is :

$$P_a=P_{a_1}+P_{a_2}=\sum_{x_1=0}^{c_1}\binom{n_1}{x_1}p^{x_1}(1-p)^{n_1-x_1}+\sum_{x_1=c_1+1}^{c_2}\left\{\left[\binom{n_1}{x_1}p^{x_1}(1-p)^{n_1-x_1}\right]\times\left[\sum_{x_2=0}^{c_2-x_1}\binom{n_2}{x_2}p^{x_2}(1-p)^{n_2-x_2}\right]\right\} \tag{5}$$

Probability of rejection on the first sample is:

$$1-P_{a1}=1-\sum_{x_1=0}^{c_1}\binom{n_1}{x_1}p^{x_1}(1-p)^{n_1-x_1}$$

The Average Outgoing Quality (AOQ) for Double Sampling Plan is obtained by combining the expected proportion of defective units in the accepted lot on the first sample is $p(N-n_1)P_{a1}$ and the proportion of defective units in the outgoing lot which is accepted on the second sample $p(N-n_1-n_2)P_{a2}$.

The combined proportion of defective units in the accepted lots is given as $p(N-n_1)P_{a1}+p(N-n_1-n_2)P_{a2}$ (Iorkegh and Osanaiye 2022).

The Average Outgoing Quality (AOQ) for rectifying double sampling plan is given as:

$$AOQ_1=\frac{p\left[P_{a_1}(N-n_1)+P_{a_2}(N-n_1-n_2)\right]}{N} \tag{6}$$

Given that it is accepted on the second sample it means that two samples $(n_1 + n_2)$ are inspected with the probability of acceptance P_{a_2} and if the whole lot N is rejected, with the probability $(1 - P_{a1} - P_{a2})$,the Average Total Inspection (ATI) is calculated as:

ATI$=n_1 P_{a_1} + (n_1 + n_2)P_{a2} + N(1 - P_{a1} - P_{a2})$ (7)

3.4 Design of Sampling Plans with specified AQL and LTPD

Acceptable Quality Level (AQL) is the small percentage of defective units in a lot that is acceptable by the consumer. Rejecting a lot with acceptable quality level results to producer risk (α) Also Lot Tolerant Percent Defective (LTPD) is the highest percentage of defective units a consumer can not tolerate in a lot. Accepting lot with this quality level results to consumer's risk(β). The Acceptable Quality Level (AQL) of 0.02 with a producer's risk $(\alpha) = 0.05$ is considered in this paper for the design of Single and Double sampling plans such that the probability of rejection of the lot given acceptable quality level should be $AQL \leq \alpha$ should be less than or equal to the value of the producer's risk(α). On the other hand, Lot Tolerant Percent Defective (LTPD) of 0.07 with consumer's risk$(\beta) = 0.1$ is also used in this design such that. the probability of acceptance of the lot given lot Tolerant Proportion Defective (LTPD) should be$LTPD \leq \beta$

3.5 Probability of Acceptance in Single Sampling Plan given AQL and LTPD Levels of quality

In the design of acceptance single sampling plan, we chose an appropriate sample size (n) and acceptance number (c). Given the probabilities of acceptance(P_a) with associated quality level$s$ AQL, and $LTPD$.Probability of lot acceptance is thus given as:

$1 - \alpha = P_a(x \leq c|n, AQL) = \sum_{x=0}^{c} \binom{n}{x} AQL^x (1 - AQL)^{n-x}$ (8)

$1 - \alpha = P_a(AQL)$ (9)

The probability of rejection at $p_1 = AQL$ or producer's risk is:

$1 - P_a(AQL) = \alpha$ (10)

On the other hand, if the lot with quality level $LTPD$ is accepted, then there is consumer's risk. Thus, the probability of accepting lot with quality level $p_2 = LTPD$ is stated below:

$\beta = \sum_{x=0}^{c} \binom{n}{x} LTPD^X (1 - LTPD)^{n-x}$

$\beta = P_a(LTPD)$ (11)

3.5 Finding Probability of Acceptance for Double Sampling Plan given AQL and LTPD levels of quality.

In the design of Double Sampling Plan, an appropriate sampling plann_1, $n_{2,}c_1$ andc_2 is found and the probabilities of acceptance at a quality level AQL and $LTPD$ is formulated. Thus, the probabilities of lot acceptance given quality level AQL and $LTPD$ are determined as shown below:

$1 - \alpha = P_{a_1} + P_{a_2} = P(x_1 \leq c_1|n_{1,}AQL) + P(x_1 + x_2 \leq c_2|c_1 < x_1 \leq c_2, n_{1,}n_{2,}AQL)$ (12)

$= \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} AQL^{x_1}(1 - AQL)^{n_1-x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} AQL^{x_1}(1 - AQL)^{n_1-x_1} \right] \times \left[\sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} AQL^{x_2} (1 - AQL)^{n_2-x_2} \right] \right\}$ (13)

$1 - \alpha = P_a(AQL)$ (14)

Producer's risk is then given as:

$1 - P_a(AQL) = \alpha$ (15)

$\beta = P_{a_1} + P_{a_2} = P(x_1 \leq c_1|n_{1,}LTPD) + P(x_1 + x_2 \leq c_2|c_1 < x_1 \leq c_2, n_{1,}n_{2,}LTPD)$ (16)

$\beta = \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} LTPD^{x_1}(1 - LTPD)^{n_1-x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} LTPD^{x_1}(1 - LTPD)^{n_1-x_1} \right] \times \left[\sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} LTPD^{x_2} (1 - LTPD)^{n_2-x_2} \right] \right\}$

Consumer's risk is given as:

$\beta = P_a(LTPD)$ (17)

3.6 Average Total Inspection (ATI) Minimization Model

A model to achieve optimal sampling plan with optimal Average Total Inspection (ATI) for both Single and Double sampling plans such that both the producer's and the consumer's risks are minimized is as given below:

Minimize: (ATI) (18)

Maximize: $P_{a(}(p)$

Subject to $1 - P_a(AQL) \leq \alpha$

$P_a(LTPD) \leq \beta$

$1 - P_a(AQL)$ is the probability of rejection of the lot at acceptable quality level, $P_a(LTPD)$ is the probability of acceptance of the lot given Lot tolerance percent defective. In addition, α and β represent producer's risk and consumer's risk respectively.

4. Findings

4.1 Effect of Sample Size $(\boldsymbol{n})$ and Acceptance Number $(\boldsymbol{c})$ on OC curve of SSP and DSP

The effect of sample size and acceptance number on the operating characteristics curve of Single Sampling Plan (SSP) and Double Sampling Plan (DSP) is as shown in the tables and figures below:

Table: 1 Operating Characteristics (OC) Curve for SSP with increasing sample size (n) and constant acceptance number.

p	$(n = 50, c = 6)$	$(n = 100, c = 6)$	$(n = 150, c = 6)$	$(n = 200, c = 6)$
	$P_a(p)$	$P_a(p)$	$P_a(p)$	$P_a(p)$
0.01	0.9999	0.9999	0.9992	0.9957

0.02(AQL)	0.9999	0.9959	0.9680	0.8957
0.03	0.9992	0.9687	0.8384	0.6063
0.04	0.9992	0.8936	0.6063	0.3083
0.05	0.9882	0.7660	0.3729	0.1237
0.06	0.9711	0.6063	0.1984	0.0413
0.07(LTPD)	0.9417	0.4442	0.0934	0.0119
0.08	0.8981	03031	0.0397	0.0030
0.09	0.8404	0.1931	0.0155	0.0007
0.10	0.7702	0.1172	0.0056	0.0001
0.11	0.6909	0.0672	0.0019	0.0000
0.12	0.6065	0.0367	0.0006	0.0000

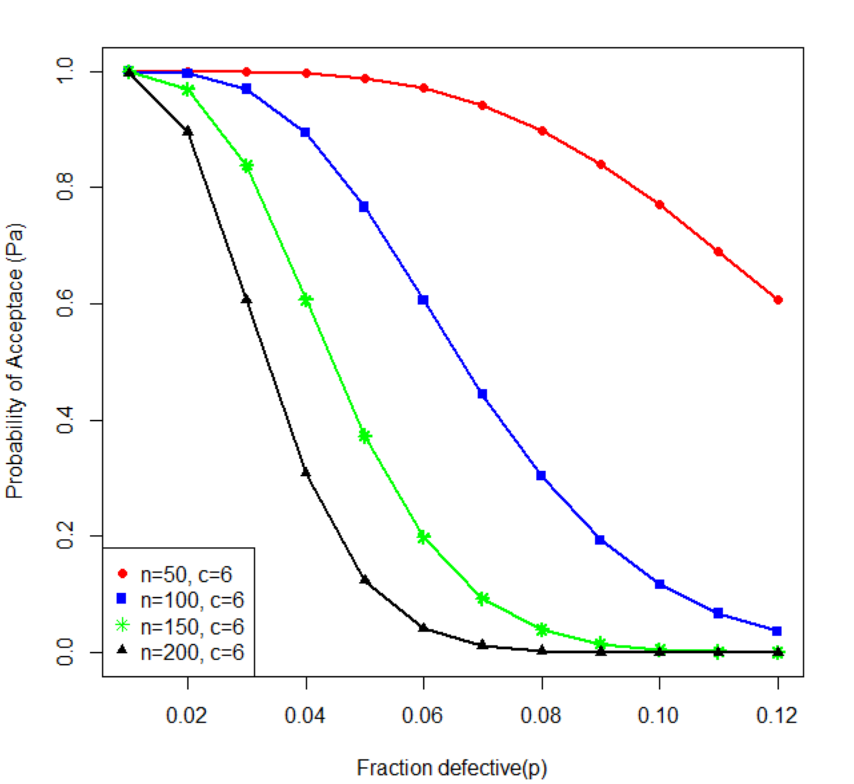

Fig.1:Effect of increase in Sample Size (n) on the OC curve of SSP

The effect of icrease in sample size on the operating characteristics (OC) curve of Single Sampling Plan (SSP) is shown in table 1 and fig.1 above. Increaseing the sample size from 50 to 200 and keeping the acceptance number constant at 6 makes it difficult for the lot to be accepted by the consumer.This is because the chances of finding more defectove units is greater when the sample size is large. From table 1 and figure 1 above it is observed that as the sample sample size increased from 50 to 200 and acceptance number c is constant at 6, the operating characteristics (OC) curve is seen to have shifted to the left incicating a decrease in the probability of acceptance of the lot thereby increasing the producer's risk but decreasing the consumer's risk.

Table: 2 Operating Characteristics (OC) Curve for SSP with increasing acceptance number (C) and constant sample size

p	$(n = 50, c = 6)$ $P_a(p)$	$(n = 50, c = 7)$ $P_a(p)$	$(n = 50, c = 8)$ $P_a(p)$	$(n = 50, c = 9)$ $P_a(p)$
0.01	0.9999	0.8999	0.9999	1.0000
0.02(AQL)	0.9999	0.9999	0.9989	0.9999
0.03	0.9992	0.9998	0.9899	0.9999
0.04	0.9963	0.9992	0.9898	0.9999
0.05	0.9882	0.9968	0.9992	0.9998
0.06	0.9711	0.9906	0.9973	0.9993
0.07(LTPD)	0.9417	0.9779	0.9927	0.9978
0.08	0.8981	0.9562	0.9833	0.9944
0.09	0.8404	0.9232	0.9672	0.9875
0.10	0.7702	0.8779	0.9421	0.9755
0.11	0.6909	0.8207	0.9068	0.9565
0.12	0.6065	0.7533	0.8607	0.9292

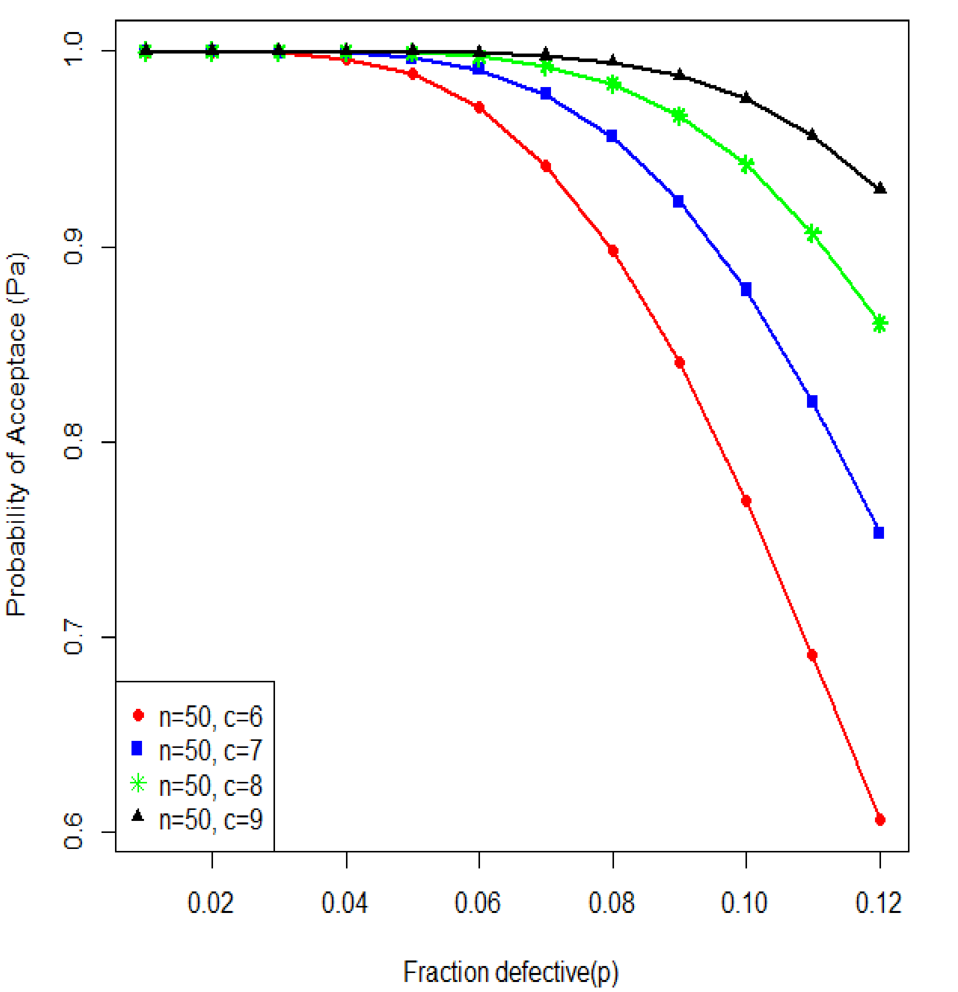

Fig.2:Effect of increase in Acceptace number (C) on the OC curve of SSP

The effect of increasing the acceptance number(c) but keeping the sample size (n) constant decrease the producer's risk but increase the consumer's risk. This is because by increasing the acceptance number and keeping the sample size constant makes it easy for a lot to be accepted thereby increasing the the consumer's risk.

It can be seen in the table 2 and fig. 2 above that, as the acceptance number is increased from 6 to 9 and the sample size is constant at 50, the operating characteristics curve is shifted to the right indicating increase in the probability of acceptance of the lot thus increasing the consumer's risk but decreasing the producer's risk.

4.2 Effect of Sample Size $(\boldsymbol{n})$ on the Operating Characteristics (OC) curve of DSP

Table 3 Operating Characteristics (OC) Curve for DSP with increasing sample size (n) and constant acceptance number

p	$\binom{n_1 = 66, n_2 = 132}{c_1 = 1, c_2 = 8}$	$\binom{n_1 = 68, n_2 = 136}{c_1 = 1, c_2 = 8}$	$\binom{n_1 = 70, n_2 = 140}{c_1 = 1, c_2 = 8}$	$\binom{n_1 = 72, n_2 = 144}{c_1 = 1, c_2 = 8}$
	$P_a(p)$	$P_a(p)$	$P_a(p)$	$P_a(p)$
0.01	0.9998	0.9998	0.9997	0.9907
0.02(AQL)	0.9832	0.9801	0.9765	0.9726
0.03	0.8717	0.8546	0.8365	0.8173
0.04	0.6394	0.6074	0.5755	0.5437
0.05	0.3859	0.3526	0.3211	0.2916
0.06	0.9999	0.1756	0.1538	0.1345
0.07(LTPD)	0.0941	0.0799	0.0678	0.0576
0.08	0.0426	0.0353	0.0294	0.0245
0.09	0.0194	0.0159	0.0131	0.0108
0.10	0.0092	0.0074	0.0060	0.0049
0.11	0.0045	0.0036	0.0029	0.0023
0.12	0.0022	0.0018	0.0014	0.0011

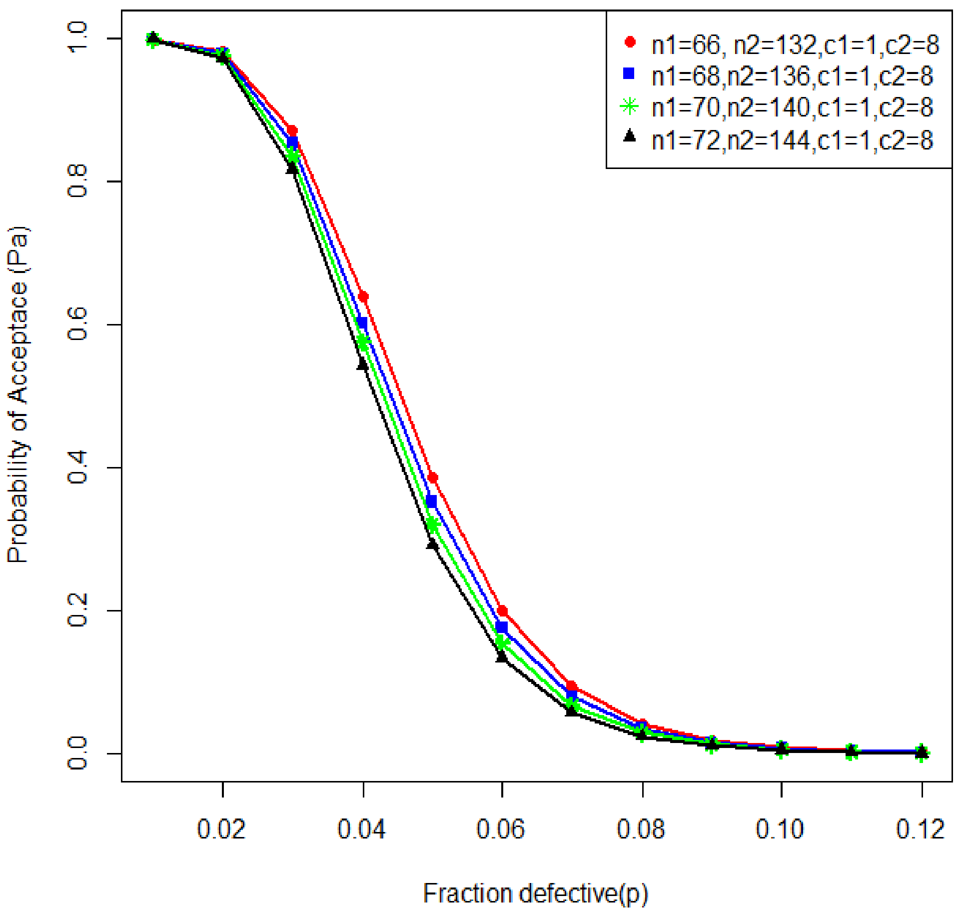

Fig.3:Effect of Increasing Sample size on the OC curve of DSP

Table 3 and fig.3 above show the effect of increase in sample size and keeping acceptace number constant on the operating characteristics (OC) curve of Double Sampling Plan (DSP). Increaseing the sample size $n1 = 66$, n2=132 to $n1 = 72, n2 = 144$ and keepig the acceptance numbers $(c1) = 1$ and $C2 = 8$ constant increases producer's risk but decreases the consumer's risk. This is because by increasing the sample size makes it easy for the consumer to find defective units in the sample thus decreasing the combined probabilities of acceptance of the lot . Therefore producer's risk is increased while the consumer's risk is reduced.
It can be seen in the table 3 and figure 3 above that as the sample sample size increased from and and acceptance number is constant , the operating characteristics (OC) curve of Double Sampling Plan (DSP) shifted to the left incicating a decrease in the combined probabilities of acceptance of the lot leading to increase in the producer's risk

4.3 Effect of Acceptance Number $(\boldsymbol{c})$ on the Operating Characteristics (OC) curve of DSP

Table: 4 Operating Characteristics (OC) Curve for DSP with increasing acceptance number (c) and constant sample size

p	$\begin{pmatrix} n_1 = 66, n_2 = 132 \\ c_1 = 1, c_2 = 8 \end{pmatrix}$	$\begin{pmatrix} n_1 = 66, n_2 = 132 \\ c_1 = 2, c_2 = 10 \end{pmatrix}$	$\begin{pmatrix} n_1 = 66, n_2 = 132 \\ c_1 = 3, c_2 = 11 \end{pmatrix}$	$\begin{pmatrix} n_1 = 66, n_2 = 132 \\ c_1 = 4, c_2 = 12 \end{pmatrix}$
	$P_a(p)$	$P_a(p)$	$P_a(p)$	$P_a(p)$
0.01	0.9999	1.0000	1.0000	1.0000
0.02(AQL)	0.9873	0.9987	0.9795	0.9999
0.03	0.9001	0.9778	0.9887	0.9964
0.04	0.7092	0.8934	0.9306	0.9708
0.05	0.4848	0.7334	0.7945	0.8979
0.06	0.3006	0.5421	0.6050	0.7569
0.07(LTPD)	0.1773	0.3709	0.4173	0.5919
0.08	0.1028	0.2422	0.2689	0.4343
0.09	0.0593	0.1542	0.1668	0.3052
0.10	0.0340	0.0966	0.1017	0.0284
0.11	0.0193	0.0597	0.0614	0.1393
0.12	0.0109	0.0362	0.0368	0.0912

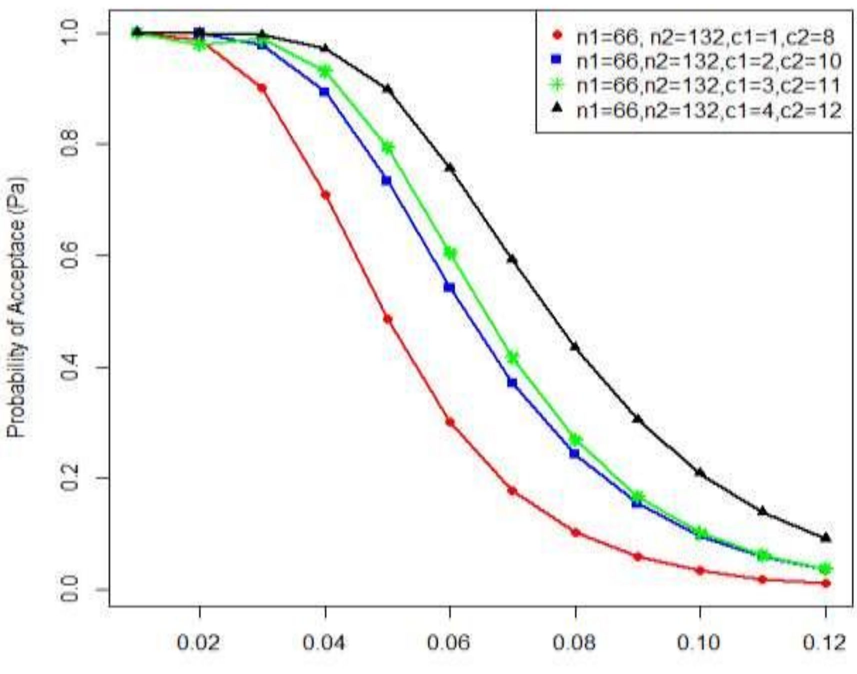

Fig.4:Effect of Acceptance number(C) on the OC curve of DSP

The effect of acceptance number(c) on the operating characteristics (OC) curve of Double Sampling Plan (DSP) is shown in table 4 and fig.4 above. Increasing the acceptance numbers $c1 = 2$ to $c1 = 4, c2 = 8$ to $c2 = 12$ and keeping the sample sizes $n1 = 66, n2 = 132$ constant increase the consumer's risk but decrease the producer's risk. This is because by increasing the acceptance numbers the chances of finding the maximum defective units in a sample that allow the rejection of the lot is reduced hence the consumer's risk. However, there is decrease in the producer's risk because the combined probabilities of acceptance of the lot is high Table 4 and fig. 4 above indicate that, as the acceptance number is increased the operating characteristics curve of Double Sampling plan (DSP) is shifted to the right indicating increase in the combined probabilities of acceptance of the lot

4.4 Determination of optimal sampling plans for Single Sampling and Double Sampling Plans

Optimal sampling plans for Single Sampling Plan (SSP) and Double Sampling Plan (DSP) would provide the desired sample size, minimize the Average Total Inspection (ATI) and also keep both $1 - P_a(AQL)$ and $P_a(LTPD)$ within the required threshold of ≤0.05 and ≤0.10 respectively is computed as shown below.

Table 5 Single Sampling Plans (SSP) with the parameters ($N = 1000, AQL = 0.02, LTPD = 0.07$, α=0.05, β=0.1, $AOQL = 0.03$ with $n \leq 205$)

n	c	AOQ	ATI	$1 - P_a(AQL)$	$P_a(LTPD)$	$P_a(p)$
196	7	0.0184	386.83	0.0448	0.0322	0.7627
196	8	0.0208	306.47	0.0180	0.0642	0.8626
197	7	0.0183	390.88	0.0459	0.0309	0.7586
197	8	0.0207	309.74	0.0185	0.0620	0.8596
198	7	0.0182	394.95	0.0470	0.0297	0.7544
198	8	0.0206	313.04	0.0190	0.0598	0.8566
199	7	0.0180	399.03	0.0482	0.0285	0.7503
199	8	0.0205	316.35	0.0196	0.0577	0.8535
200	7	0.0179	403.12	0.0493	0.0274	0.7461
200	8	0.0204	319.68	0.0202	0.0556	0.8504
201	8	0.0203	323.03	0.0208	0.0537	0.8473
201	9	0.0220	267.19	0.0077	0.0979	0.9172
202	8	0.0202	326.40	0.0214	0.0518	0.8441
202	9	0.0219	269.78	0.0080	0.0947	0.9151
203	8	0.0201	329.79	0.0220	0.0499	0.8409
203	9	0.0218	272.39	0.0083	0.0917	0.9129
204	8	0.0200	333.19	0.0226	0.0481	0.8377
204	9	0.0217	275.03	0.0085	0.0888	0.9108
205	8	0.0199	336.62	0.0232	0.0464	0.8344
205	9	0.0217	277.68	0.0088	0.0859	0.9086

Table 5 shows different sampling plans generated for Single Sampling Plan (SSP) with Average Total Inspection (ATI), probability of rejection of the lot given acceptable quality level $1 - P_a(AQL)$, probability of acceptance given Lot Tolerant Percent defective units $P_a(LTPD)$ and probability of acceptance of the lot $P_a(p)$ for all the sampling plans. However, the optimal sampling plan is obtained at $n = 201$,$C = 9$ with a minimum Average Total Inspection (ATI)) of 267.19 and higher probability of acceptance $P_a(p)$ of 0.9172 satisfying both the producer 's risk of $1 - P_a(AQL) \leq 0.05$ and consumer's risk of $P_a(LTPD) \leq 0.1$ respectively.

Table 6: Double Sampling Plan (DSP) Plans with the given parameters ($N = 1000, AQL = 0.02, LTPD = 0.07$, α=0.05, β=0.1, $AOQL = 0.03$,withn_1 and $n_2 \leq 205$)

n_1	n_2	c_1	c_2	AOQ	ATI	$1 - P_a(AQL)$	$P_a(LTPD)$	$P_a(p)$
66	132	1	8	0.0226	247.16	0.0168	0.0941	0.8717
67	134	1	8	0.0223	256.78	0.0183	0.0867	0.8632
68	136	1	8	0.0222	266.57	0.0199	0.0799	0.8546
69	138	1	8	0.0217	276.53	0.0216	0.0736	0.8456
70	140	1	8	0.0214	286.66	0.0235	0.0678	0.8365
71	142	1	8	0.0211	296.94	0.0254	0.0625	0.8270
72	144	1	8	0.0208	307.35	0.0274	0.0576	0.8173
73	146	1	8	0.0205	317.90	0.0295	0.0530	0.8075
74	148	1	8	0.0201	328.57	0.0318	0.0489	0.7973
75	150	1	8	0.0198	339.35	0.0342	0.0450	0.7870
76	152	1	8	0.0195	350.23	0.0367	0.0415	0.7465
77	154	1	8	0.0192	361.19	0.0393	0.0383	0.7658
77	154	2	8	0.0215	282.40	0.0300	0.0953	0.8146
78	156	1	8	0.0188	372.22	0.0420	0.0353	0.7550
78	156	2	8	0.0212	291.74	0.0321	0.0901	0.8057
79	158	1	8	0.0185	383.32	0.0448	0.0326	0.7439
79	158	2	8	0.0210	301.17	0.0343	0.0852	0.7966
80	160	1	8	0.0182	394.48	0.0478	0.0301	0.7328
80	160	2	8	0.0207	310.68	0.0366	0.0805	0.7874
81	162	2	8	0.0204	320.27	0.0390	0.0761	0.7780
82	164	2	8	0.0201	329.92	0.0415	0.0719	0.7686
83	166	2	8	0.0198	339.63	0.0441	0.0680	0.759
95	190	3	10	0.0221	263.69	0.0175	0.0972	0.8488
96	192	3	10	0.0219	271.07	0.0187	0.0926	0.8419
96	192	3	11	0.0229	237.32	0.0093	0.0971	0.8891
97	194	3	10	0.0216	278.10	0.0200	0.0882	0.8349
97	194	3	11	0.0227	244.10	0.0100	0.0923	0.8835
98	196	3	10	0.0214	286.10	0.0213	0.0841	0.8277
98	196	3	11	0.0225	250.98	0.0108	0.0878	0.8775

Table 6 shows different sampling plans generated for Double Sampling Plan (DSP) with Average Total Inspection (ATI), combined probability of rejection of the lot given acceptable quality level $1 - P_a(AQL)$, combined probability of acceptance given Lot Tolerant Percent defective units $P_a(LTPD)$ and combined probability of acceptance of the lot $P_a(p)$ for all the sampling plans. However, the optimal sampling plan is obtained at $n1 = 96, n2 = 192, c1 = 3, c2 = 11$, with a minimum Average Total Inspection (ATI) of 237.32 and higher probability of acceptance $P_a(p)$ of 0.8891 satisfying both the producer 's of $1 - P_a(AQL) \leq 0.05$ and consumer's risk of $P_a(LTPD) \leq 0.1$ respectively.

4.5 Average Total Inspection (ATI) for optimal SSP and DSP

The Average Total Inspection (ATI) at the optimal SSP and DSP is compared and shown below:

Table 7: Effect of fraction defective (p) on the Average Total Inspection (ATI) in optimal Single Sampling Plan (SSP) and Double Sampling Plan (DSP)

Optimal Single Sampling Plan (SSP) n=201, c=9		Optimal Double Sampling plan (DSP) $n_1 = 96, c_1$=3,$n_2 = 192, c_2 = 11$	
p	ATI	p	ATI
0.01	201.03	0.01	99.10
0.02	207.18	0.02	126.99
0.03	267.19	0.03	237.32
0.04	429.41	0.04	454.21
0.05	641.79	0.05	676.94
0.06	816.99	0.06	829.04
0.07	921.86	0.07	913.65

0.08	971.36	0.08	957.32
0.09	990.79	0.09	979.38
0.10	997.35	0.10	990.31
0.11	999.30	0.11	999.59
0.12	999.80	0.12	998.05

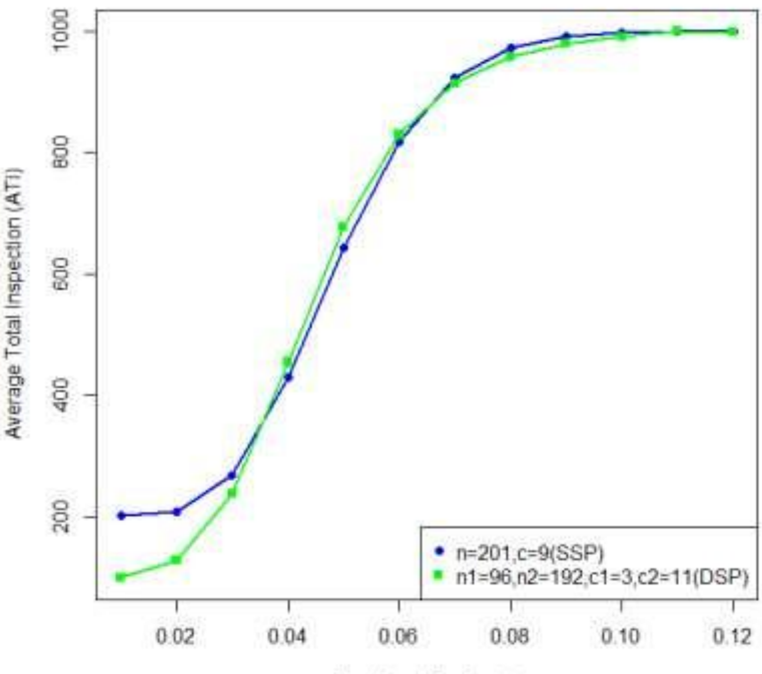

Fig.5:Average Total Inspection ATI of optimal SSP and DSP

In fig. 5 above, the effect of fraction defective units of the Average Total Inspection (ATI) of the optimal sampling plans for both Single and Double sampling plans is compared. The Average Total Inspection (ATI) for both plans are seen to have increased and converged as the fraction defective units increased. However, the Average Total Inspection (ATI) for optimal Single sampling plan is generally higher than that of the optimal Double sampling plan due to the larger sample size of the optimal Single sampling plan.

5. Conclusion and Recommendations

In this paper an economic cost model is proposed to minimize Average Total Inspection (ATI) in acceptance sampling inspection while satisfying predetermined producer's and consumer's risk requirements.

Findings indicate that the optimal sampling plans for Single Sampling Plan (SSP) and Double Sampling Plan (DSP) reduced Average Total Inspection (ATI) and provided protection for producer and consumer against risk

The study therefore significant as it provides practical guidelines in selecting cost-effective sampling inspection plans for quality control practitioners. The results of this study can be used by producers and consumers and other stakeholders involved in a supply chain to reduce inspection cost while maintaining an acceptable level of quality products.

Future studies on the proposed model to improve the optimization of the sampling plans can be carried out using Machine-learning techniques

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