

Exceptional Collections, Beilinson's Resolution of the Diagonal, and Semiorthogonal Decompositions of Derived Categories

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ARTICLE INFORMATION	ABSTRACT
Article history: Published: August 2026	We give a self-contained account of the theory of exceptional collections and semiorthogonal decompositions in $D^b(\mathcal{X})$, the bounded derived category of coherent sheaves on a smooth projective variety $\mathcal{X}$. After developing the formalism of admissible subcategories and Serre functors, we prove Beilinson's theorem that $D^b(\mathbb{P}^n) = \langle \mathcal{O}, \mathcal{O}(1), \dots, \mathcal{O}(n) \rangle$ via the Koszul resolution of the diagonal. We then treat mutations and the braid group action, Orlov's projective bundle and blow-up formulas, Kapranov's collections on quadrics, and the obstruction to exceptional objects on varieties with trivial canonical bundle. A concluding section surveys the Kuznetsov component of a cubic fourfold and its conjectural relation to rationality, including the categorical Torelli theorem. The article is expository.
Keywords: Derived Categories Exceptional Collections Semiorthogonal Decompositions Beilinson's Theorem Cubic Fourfolds	

1. Introduction

Let $\mathcal{X}$ be a smooth projective variety over a field $\mathbb{k}$, and let $D^b(\mathcal{X})$ denote the bounded derived category of coherent sheaves on $\mathcal{X}$. Since the work of Mukai on abelian varieties [13] and of Beilinson on projective space [2], the category $D^b(\mathcal{X})$ has emerged as a fundamental invariant of $\mathcal{X}$: fine enough to detect subtle birational and Hodge-theoretic information, yet flexible enough to admit equivalences between the derived categories of non-isomorphic varieties.

A central organizing principle is the notion of a semiorthogonal decomposition $D^b(\mathcal{X}) = \langle \mathcal{A}_1, \dots, \mathcal{A}_m \rangle$, a filtration-like structure that breaks the derived category into simpler pieces. The strongest instance is a full exceptional collection, in which each piece is equivalent to the derived category of a point. Beilinson's theorem, the first and paradigmatic example, asserts that

$$D^b(\mathbb{P}^n) = \langle \mathcal{O}, \mathcal{O}(1), \dots, \mathcal{O}(n) \rangle,$$

so that the entire homological algebra of coherent sheaves on projective space is encoded in the endomorphism algebra of $\bigoplus_{i=0}^n \mathcal{O}(i)$, a finite-dimensional algebra.

We develop the general formalism of exceptional objects, admissible subcategories, and Serre functors (Sections 2–3), with proofs of the foundational lemmas. Then, we prove Beilinson's theorem via the Koszul resolution of the diagonal and develop the classical extensions: mutations and the braid group action, Orlov's theorems on projective bundles and blow-ups, and Kapranov's collections on quadrics (Section 4). Finally, we survey the modern frontier: Kuznetsov components, categorical Torelli theorems, and the interaction between semiorthogonal decompositions and rationality of cubic fourfolds (Section 4.5), closing with open problems (Section 5).

Throughout, $\mathbb{k}$ is algebraically closed of characteristic zero, all varieties are smooth and projective, and all functors between derived categories are implicitly derived. We write $\mathrm{Hom}^*(E, F) = \bigoplus_i \mathrm{Hom}(E, F[i])[-i]$ for the graded Hom-complex.

2. Preliminaries on Triangulated Categories

All triangulated categories below are $\mathbb{k}$ -linear and Π -finite, meaning $\dim_{\mathbb{k}} \bigoplus_i \mathrm{Hom}(E, F[i]) < \infty$ for all objects E, F ; this holds for $D^b(\mathcal{X})$ when $\mathcal{X}$ is smooth and projective.

2.1 Serre functors and Serre–Grothendieck duality

A Serre functor on a Π -finite $\mathbb{k}$ -linear triangulated category $\mathcal{T}$ is an autoequivalence $S: \mathcal{T} \rightarrow \mathcal{T}$ together with bifunctorial isomorphisms $\mathrm{Hom}(E, F) \cong \mathrm{Hom}(F, SE)^\vee$ for all $E, F \in \mathcal{T}$. When it exists, a Serre functor is unique up to canonical isomorphism [4]. The fundamental example is Serre–Grothendieck duality.

Theorem 2.1 ([4]). *If $\mathcal{X}$ is smooth projective of dimension n , then $S_{\mathcal{X}} := (-) \otimes \omega_{\mathcal{X}}[n]$ is a Serre functor on $D^b(\mathcal{X})$.*

2.2 Admissible subcategories and orthogonals

A full triangulated subcategory $\mathcal{A} \subset \mathcal{T}$ is right admissible if the inclusion $i: \mathcal{A} \hookrightarrow \mathcal{T}$ admits a right adjoint $j^!$, left admissible if it admits a left adjoint j^* , and admissible if both hold. For a full subcategory $\mathcal{A} \subset \mathcal{T}$ we define its orthogonals

$$\mathcal{A}^\perp = \{T : \mathrm{Hom}(\mathcal{A}, T) = 0 \ \forall \mathcal{A} \in \mathcal{A}\}, \quad {}^\perp \mathcal{A} = \{T : \mathrm{Hom}(T, \mathcal{A}) = 0 \ \forall \mathcal{A} \in \mathcal{A}\}.$$

The following lemma is the engine behind everything that follows.

Lemma 2.2. *Let $\mathcal{A} \subset \mathcal{T}$ be right admissible. Then for every $\mathcal{T} \subset \mathcal{T}$ there is a distinguished triangle $i^!T \rightarrow T \rightarrow C \rightarrow i^!T[1]$ with $C \in \mathcal{A}^\perp$. Consequently $\mathcal{T}$ is generated by $\mathcal{A}$ and $\mathcal{A}^\perp$.*

Proof. Complete the counit $\varepsilon_T: i^!T \rightarrow T$ to a distinguished triangle with cone C_T . For any $A \in \mathcal{A}$, applying $\text{Hom}(iA, -)$ yields a long exact sequence in which the map induced by ε_T is identified, by adjunction, with the isomorphism $\text{Hom}_{\mathcal{A}}(A, i^!T[j]) \xrightarrow{\sim} \text{Hom}(iA, T[j])$ for every j . Hence $\text{Hom}(iA, C[j]) = 0$ for all j , i.e. $C \in \mathcal{A}^\perp$. $\square$

3. Semiorthogonal Decompositions and Exceptional Collections

3.1 Semiorthogonal decompositions

A semiorthogonal decomposition of $\mathcal{T}$, written $\mathcal{T} = \langle \mathcal{A}_1, \dots, \mathcal{A}_m \rangle$, is a sequence of full triangulated subcategories such that (i) $\text{Hom}(A_j, A_i) = 0$ for $A_i \in \mathcal{A}_i, A_j \in \mathcal{A}_j, i < j$, whenever $i < j$ and (ii) every $T \in \mathcal{T}$ admits a chain $0 = T_m \rightarrow T_{m-1} \rightarrow \dots \rightarrow T_0 = T$ whose cones satisfy $\text{cone}(T_i \rightarrow T_{i-1}) \in \mathcal{A}_i$. Condition (i) forces the filtration in (ii) to be unique and functorial.

3.2 Exceptional objects and admissibility

An object $E \in \mathcal{T}$ is exceptional if $\text{Hom}(E, E[j]) = \mathbb{k}$ for $j = 0$ and vanishes otherwise. A sequence $(E_1, \dots, E_m)$ of exceptional objects is an exceptional collection if $\text{Hom}(E_j, E_i[\ell]) = 0$ for all ℓ whenever $i < j$. It is full if the smallest thick triangulated subcategory containing all E_i is $\mathcal{T}$, and strong if in addition $\text{Hom}(E_i, E_j[\ell]) = 0$ for all $\ell \neq 0$. For exceptional E , the subcategory $\langle E \rangle$ is equivalent to $D^b(\mathbb{k})$ via $V^* \mapsto V^* \otimes E$.

Lemma 3.1. *If $E \in \mathcal{T}$ is exceptional, then $\langle E \rangle$ is admissible and there is a semiorthogonal decomposition $\mathcal{T} = \langle E^\perp, E \rangle$.*

Proof. For $T \in \mathcal{T}$ define the evaluation triangle $\text{Hom}^*(E, T) \otimes E \xrightarrow{\text{ev}} T \rightarrow C_T$. Applying $\text{Hom}(E, -)$ and using exceptionality of E , the induced map $\text{Hom}^*(E, T) \otimes \text{Hom}^*(E, E) \rightarrow \text{Hom}^*(E, T)$ is an isomorphism, so $C_T \in E^\perp$. Thus $T \mapsto \text{Hom}^*(E, T) \otimes E$ is right adjoint to the inclusion of $\langle E \rangle$, giving right admissibility and, by Lemma 2.2, the decomposition $\mathcal{T} = \langle E^\perp, E \rangle$. Left admissibility is dual, via the coevaluation triangle. $\square$

Iterating Lemma 3.1 shows that an exceptional collection $(E_1, \dots, E_m)$ induces a semiorthogonal decomposition $\mathcal{T} = \langle \mathcal{A}, E_1, \dots, E_m \rangle$ with $\mathcal{A} = \{T : \text{Hom}(E_i, T[\ell]) = 0 \forall i, \ell\}$; the collection is full iff $\mathcal{A} = 0$. When the collection is full and strong, Bondal [3] proved $\mathcal{T} \simeq D^b(\text{mod-}A)$ where $A = \text{End}(\bigoplus_i E_i)$ is a finite-dimensional algebra of finite global dimension; this tilting perspective links the geometry of X to the representation theory of quivers with relations.

3.3 The Calabi–Yau obstruction

The existence of exceptional objects is strongly obstructed by the canonical bundle.

Proposition 3.2. *Let X be smooth projective of dimension $n \geq 1$ with $\omega_X \simeq \mathcal{O}_X$ (e.g. an abelian variety, K3 surface, or strict Calabi–Yau). Then $D^b(X)$ contains no exceptional objects and admits no nontrivial semiorthogonal decomposition.*

Proof. If E were exceptional, Theorem 2.1 would give $\text{Hom}(E, E[n]) = \text{Hom}(E, S_X E) \cong \text{Hom}(E, E)^\vee = \mathbb{k} \neq 0$, contradicting exceptionality since $n \neq 0$. For the second claim, suppose $D^b(X) = \langle \mathcal{A}, \mathcal{B} \rangle$ with both pieces nonzero. For $A \in \mathcal{A}, B \in \mathcal{B}$ semiorthogonality gives $\text{Hom}(B, A[\ell]) = 0$, while Serre duality gives $\text{Hom}(A, B[\ell]) \cong \text{Hom}(B, A[n-\ell])^\vee = 0$. Hence the decomposition is completely orthogonal, $D^b(X) = \mathcal{A} \oplus \mathcal{B}$; but $D^b(X)$ is indecomposable for X connected (the structure sheaf, then every point sheaf, then all of $D^b(X)$ lies in one summand), so $\mathcal{B} = 0$. $\square$

Consequently a smooth projective curve C admits a full exceptional collection if and only if $C \simeq \mathbb{P}^1$: for $g(C) \geq 1$ either $\omega_C \simeq \mathcal{O}_C$, or $H_1(C, \mathbb{Z})$ contains the divisible group $\mathbb{Q}$ and so cannot be free of finite rank.

4. Beilinson's Theorem and Its Extensions

Fix a vector space V of dimension $n+1$ and let $\mathbb{P} = \mathbb{P}(V) = \mathbb{P}^n$, so that $H^0(\mathbb{P}, \mathcal{O}(1)) = V^\vee$. Recall the Euler sequence

$$0 \rightarrow \Omega_{\mathbb{P}}^1(1) \rightarrow V^\vee \otimes \mathcal{O}_{\mathbb{P}} \rightarrow \mathcal{O}_{\mathbb{P}}(1) \rightarrow 0.$$

Theorem 4.1 (Beilinson [2]). *The collection $(\mathcal{O}, \mathcal{O}(1), \dots, \mathcal{O}(n))$ is a full strong exceptional collection in $D^b(\mathbb{P}^n)$. Likewise $(\Omega^n(n), \Omega^{n-1}(n-1), \dots, \Omega^1(1), \mathcal{O})$ is a full exceptional collection.*

4.1 The resolution of the diagonal

Step 1 (Exceptionality). By the standard computation of the cohomology of line bundles on $\mathbb{P}^n$ [7], one has $\text{Hom}(\mathcal{O}(i), \mathcal{O}(j)[\ell]) = H^\ell(\mathbb{P}^n, \mathcal{O}(j-i))$, which is $\mathbb{k}$ if $j-i = \ell \geq 0$ and vanishes for $-n \leq j-i \leq n$ otherwise. Thus each $\mathcal{O}(i)$ is exceptional, $\text{Hom}^*(\mathcal{O}(j), \mathcal{O}(i)) = 0$ for $i < j$, and all Hom's are concentrated in degree 0; the collection is exceptional and strong.

Step 2 (Resolution of the diagonal). Let $p, q: \mathbb{P} \times \mathbb{P} \rightarrow \mathbb{P}$ be the projections and $\Delta \subset \mathbb{P} \times \mathbb{P}$ the diagonal. Consider the rank- n bundle $\mathcal{E} := \mathcal{O}_{\mathbb{P}}(1) \boxtimes T_{\mathbb{P}}(-1)$.

Lemma 4.2. *There is a canonical section $s \in H^0(\mathbb{P} \times \mathbb{P}, \mathcal{E})$ whose zero locus is exactly the diagonal Δ , scheme-theoretically, with $\text{codim } \Delta = \text{rk } \mathcal{E} = n$.*

Proof. By Künneth and the twisted Euler sequence (which gives $H^0(T(-1)) = V, H^i(T(-1)) = 0$), $H^0(\mathbb{P} \times \mathbb{P}, \mathcal{E}) = V^\vee \otimes V = \text{End}(V)$; let s correspond to id_V . At a point $([L], [M])$ with quotients $\phi: V \rightarrow L, \psi: V \rightarrow M$, the value of s is the composite $\ker \psi \hookrightarrow V \xrightarrow{\phi} L$, which vanishes iff $\ker \psi \subseteq \ker \phi$, i.e. $[L] = [\psi(\ker \psi)]$. A local computation shows the vanishing is transverse, so the zero scheme is the reduced diagonal of codimension n . $\square$

Since s is a regular section, its Koszul complex resolves $\mathcal{O}_\Delta$:

$$0 \rightarrow \Lambda^n \mathcal{E}^\vee \rightarrow \cdots \rightarrow \Lambda^2 \mathcal{E}^\vee \rightarrow \mathcal{E}^\vee \rightarrow \mathcal{O}_{\mathbb{P}^n \times \mathbb{P}^n} \rightarrow \mathcal{O}_\Delta \rightarrow 0,$$

and since $\mathcal{E}^\vee = \mathcal{O}(-1) \boxtimes \Omega^1(1)$, the exterior powers factor as $\Lambda^i \mathcal{E}^\vee = \mathcal{O}(-i) \boxtimes \Omega^i(1)$ for $0 \leq i \leq n$.

Step 3 (Dévissage). Every $F \in D^b(\mathbb{P}^n)$ satisfies $F \cong q_*(p^*F \otimes \mathcal{O}_\Delta)$. Replacing $\mathcal{O}_\Delta$ by the Koszul resolution exhibits F as an iterated cone of the complexes $H^*(\mathbb{P}^n, F(-i)) \otimes \Omega^i(1)$, $i = 0, \dots, n$; concretely one obtains the Beilinson spectral sequence

$$E_1^{-i,j} = H^j(\mathbb{P}^n, F(-i)) \otimes \Omega^i(1) \implies \mathcal{H}^{j-i}(F).$$

Hence $\Omega(n), \dots, \mathcal{O}$ generate $D^b(\mathbb{P}^n)$, proving fullness of the second collection; dualizing and twisting by $\mathcal{O}(n)$ shows $(\mathcal{O}, \dots, \mathcal{O}(n))$ generates, completing the proof of Theorem 4.1. Bondal's theorem then identifies $D^b(\mathbb{P}^n) \simeq D^b(\text{mod-}A)$ with A the Beilinson quiver algebra; for $n = 1$ this is the Kronecker quiver, recovering the classical description of sheaves on $\mathbb{P}^1$.

4.2 Mutations and the braid group action

Exceptional collections are far from unique. Given an exceptional pair $\{E, F\}$, the left and right mutations $L_{E,F}$ and $R_{E,F}$ are defined by the triangles

$$L_E F[-1] \rightarrow \text{Hom}^\bullet(E, F) \otimes E \xrightarrow{\text{ev}} F \rightarrow L_E F, \quad R_F E \rightarrow E \rightarrow \text{Hom}^\bullet(E, F)^\vee \otimes F \rightarrow R_F E[1].$$

Bondal [3] proved that $\{L_{E,F}, F\}$ and $\{F, R_{E,F}\}$ are again exceptional pairs generating $\{E, F\}$, and that $R_E L_E \simeq \text{id}$. For a full exceptional collection of length n , the operations $\sigma_i(E_1, \dots, E_n) = (E_1, \dots, L_{E_i} E_{i+1}, E_{i+1}, \dots)$ satisfy the braid relations $\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$, so the braid group B_n acts on the set of full exceptional collections, commuting with shifts. On Γ^d the ranks of a normalized strong collection satisfy the Markov equation $a^2 + b^2 + c^2 = 3abc$, and all solutions arise from $\{1, \dots, 1\}$ by mutation [15].

4.3 Orlov's projective bundle and blow-up formulas

Semiorthogonal decompositions propagate along the basic surgeries of birational geometry [14].

Theorem 4.3 (Projective bundle [14]). Let $\mathcal{F}$ be a rank- r bundle on X and $\pi: \mathbb{P}_X(\mathcal{F}) \rightarrow X$ the associated projective bundle with relative hyperplane class $\mathcal{O}_\pi(1)$. Each functor $\Phi_k = \pi^*(-) \otimes \mathcal{O}_\pi(k)$ is fully faithful, and $D^b(\mathbb{P}_X(\mathcal{F})) = \langle \Phi_0 D^b(X), \dots, \Phi_{r-1} D^b(X) \rangle$.

Theorem 4.4 (Blow-up [14]). Let $Y \subset X$ be smooth of codimension $r \geq 2$, $\rho: \text{Bl}_Y X \rightarrow X$ the blow-up with exceptional divisor $E = \mathbb{P}_Y(N_{Y/X})$. Then ρ^* and the functors $\Psi_k = j_* (\pi^*(-) \otimes \mathcal{O}_E(kE))$ are fully faithful, and $D^b(\text{Bl}_Y X) = \langle \Psi_{-(-1)} D^b(Y), \dots, \Psi_{-1} D^b(Y), \rho^* D^b(X) \rangle$.

Theorem 4.4 is the categorical shadow of the motivic blow-up formula and motivates the philosophy that the interesting part of D^b should behave like a birational invariant. With Theorem 4.3 it produces full exceptional collections on iterated blow-ups of Γ^d ; whether one can always use line bundles was resolved negatively by Efimov [5].

4.4 Quadrics and spinor bundles

Let $Q = Q_n \subset \mathbb{P}^{n+1}$ be a smooth quadric of dimension n , carrying its canonical spinor bundle(s) $\mathcal{S}$ (for n odd) or $\mathcal{S}^+, \mathcal{S}^-$ (for n even), arising from the half-spin representations of $\text{Spin}(n+2)$.

Theorem 4.5 (Kapranov [10]). There are full strong exceptional collections $D^b(Q_n) = \langle \mathcal{S}, \mathcal{O}, \mathcal{O}(1), \dots, \mathcal{O}(n-1) \rangle$ for n odd, and $D^b(Q_n) = \langle \mathcal{S}^-, \mathcal{S}^+, \mathcal{O}, \mathcal{O}(1), \dots, \mathcal{O}(n-1) \rangle$ for n even.

The proof follows Beilinson's template, using a Clifford-algebra analogue of the Koszul resolution of the diagonal. The length of the collection matches $\dim_{\mathbb{Q}} \bigoplus_i H^{2i}(Q, \mathbb{Q})$, as it must, since a full exceptional collection forces K_X to be free of that rank and (in characteristic 0) forces the off-diagonal Hodge numbers to vanish. For an intersection of two quadrics, Bondal–Orlov identified a decomposition featuring the derived category of the associated hyperelliptic curve — a prototype of the residual categories of the next subsection.

4.5 Kuznetsov components and cubic fourfolds

Let $X \in \Gamma^3$ be a smooth cubic fourfold. The line bundles $\mathcal{O}, \mathcal{O}(1), \mathcal{O}(2)$ form an exceptional collection, and one defines the Kuznetsov component $\mathcal{Ku}(X) := \langle \mathcal{O}_X, \mathcal{O}_X(1), \mathcal{O}_X(2) \rangle^\perp$, $D^b(X) = \langle \mathcal{Ku}(X), \mathcal{O}_X, \mathcal{O}_X(1), \mathcal{O}_X(2) \rangle$.

Kuznetsov [11] proved that $\mathcal{Ku}(X)$ is a connected Calabi–Yau category of dimension 3: its Serre functor is the shift $[2]$, and its Hochschild homology agrees with that of a K3 surface. Thus $\mathcal{Ku}(X)$ is a noncommutative K3 surface, and for special X it is honestly commutative.

Conjecture 4.6 (Kuznetsov [11]). A smooth cubic fourfold X is rational if and only if $\mathcal{Ku}(X) \simeq D^b(S)$ for some K3 surface S .

By Hassett [8], cubic fourfolds with special Hodge classes form a countable union of Noether–Lefschetz divisors $\mathcal{L}_d$ in the 20-dimensional moduli space, and $\mathcal{Ku}(X) \simeq D^b(S)$ holds precisely for the admissible values of d (e.g. $d = 14, 26, 38, 42, \dots$). All cubic fourfolds proved rational to date lie in such divisors, consistent with Conjecture 4.6, while the very general cubic fourfold is expected to be irrational — though no single cubic fourfold has yet been proved irrational. A major advance is the construction of Bridgeland stability conditions on $\mathcal{Ku}(X)$ by Bayer–Lahoz–Macrì–Stellari [1], which makes moduli of objects in $\mathcal{Ku}(X)$ into hyperkähler varieties and yields a categorical Torelli theorem: $\mathcal{Ku}(X)$ determines the cubic fourfold.

5. Conclusion and Open Problems

5.1 Conclusion

Semiorthogonal decompositions organize the derived category of a smooth projective variety into building blocks that are

simultaneously homological, representation-theoretic, and birational in nature. Beilinson's resolution of the diagonal supplies the first and guiding example on Γ^* ; Orlov's formulas propagate such decompositions along projective bundles and blow-ups; and the Kuznetsov component packages the transcendental geometry of a cubic fourfold into a noncommutative K3 category whose commutativity is conjecturally equivalent to rationality.

5.2 Open problems

- (i) Transitivity: does the braid group action, together with shifts, act transitively on full exceptional collections of $D^b(X)$ whenever one exists? Known for Γ^1, Γ^2 and del Pezzo surfaces; open already for Γ^3 .
- (ii) Fullness and phantoms: is an exceptional collection of length $\text{rk } K_3(X)$ automatically full? No in general — phantom subcategories with vanishing K-theory and Hochschild homology exist [6].
- (iii) Rationality: prove Conjecture 4.6 in either direction, or exhibit a single irrational cubic fourfold.
- (iv) Homogeneous and toric cases: do all rational homogeneous spaces G/P , and all smooth projective toric varieties, admit full (strong) exceptional collections? Known in many families, open in general.

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